AQA Maths Pure Core 3

Past Paper Pack

2006-2013

General Certificate of Education
January 2006
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Wednesday 25 January 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Question 6 (enclosed)

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find $\frac{dy}{dx}$ when $y = \tan 3x$. *(2 marks)*

(b) Given that $y = \frac{3x+1}{2x+1}$, show that $\frac{dy}{dx} = \frac{1}{(2x+1)^2}$. *(3 marks)*

2 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to

$$\int_1^3 \frac{1}{\sqrt{1+x^3}}\, dx$$

giving your answer to three significant figures. *(4 marks)*

3 (a) (i) Given that $f(x) = x^4 + 2x$, find $f'(x)$. *(1 mark)*

(ii) Hence, or otherwise, find $\int \frac{2x^3+1}{x^4+2x}\, dx$. *(2 marks)*

(b) (i) Use the substitution $u = 2x + 1$ to show that

$$\int x\sqrt{2x+1}\, dx = \frac{1}{4}\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$ *(3 marks)*

(ii) Hence show that $\int_0^4 x\sqrt{2x+1}\, dx = 19.9$ correct to three significant figures. *(4 marks)*

4 It is given that $2\operatorname{cosec}^2 x = 5 - 5\cot x$.

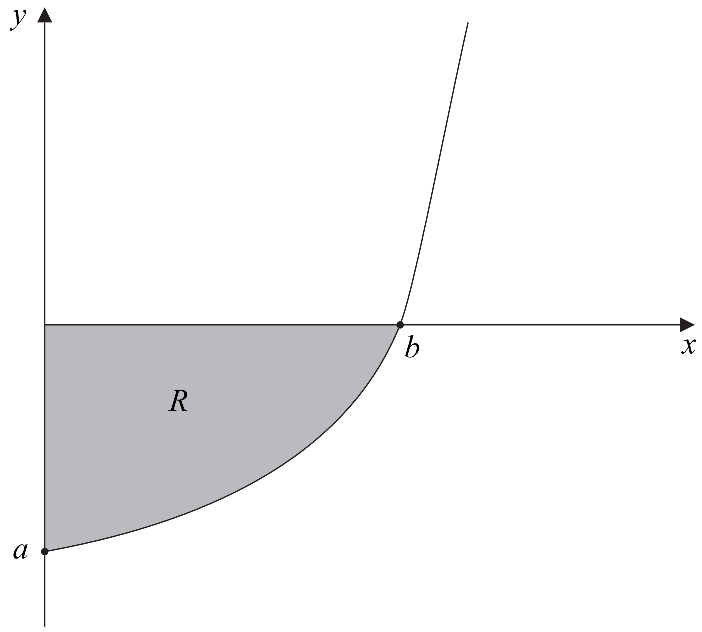
(a) Show that the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$ can be written in the form

$$2\cot^2 x + 5\cot x - 3 = 0$$ *(2 marks)*

(b) Hence show that $\tan x = 2$ or $\tan x = -\frac{1}{3}$. *(2 marks)*

(c) Hence, or otherwise, solve the equation $2\operatorname{cosec}^2 x = 5 - 5\cot x$, giving all values of x in radians to one decimal place in the interval $-\pi < x \leqslant \pi$. *(3 marks)*

5 The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.



(a) State the value of a, and show that $b = \ln 3$. *(3 marks)*

(b) Show that $y^2 = e^{4x} - 18e^{2x} + 81$. *(1 mark)*

(c) The shaded region R is rotated through 360° about the x-axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers. *(6 marks)*

(d) Sketch the curve with equation $y = |e^{2x} - 9|$ for $x \geqslant 0$. *(2 marks)*

Turn over for the next question

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve $y = x^3 + 4x - 3$ intersects the x-axis at the point A where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.0. *(2 marks)*

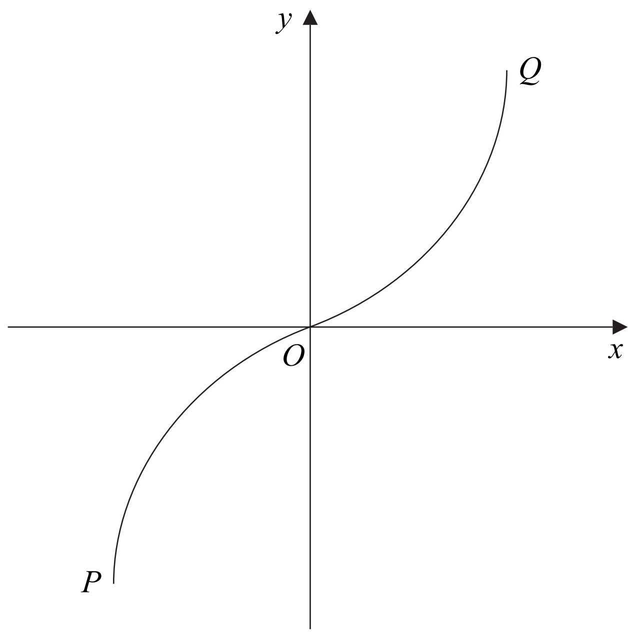
(b) Show that the equation $x^3 + 4x - 3 = 0$ can be rearranged into the form $x = \dfrac{3 - x^3}{4}$. *(1 mark)*

(c) (i) Use the iteration $x_{n+1} = \dfrac{3 - x_n{}^3}{4}$ with $x_1 = 0.5$ to find x_3, giving your answer to two decimal places. *(3 marks)*

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \dfrac{3 - x^3}{4}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(3 marks)*

7 (a) The sketch shows the graph of $y = \sin^{-1} x$.



Write down the coordinates of the points P and Q, the end-points of the graph. *(2 marks)*

(b) Sketch the graph of $y = -\sin^{-1}(x - 1)$. *(3 marks)*

8 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^2 \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x+2} \qquad \text{for real values of } x, \quad x \neq -2$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 4$. *(4 marks)*

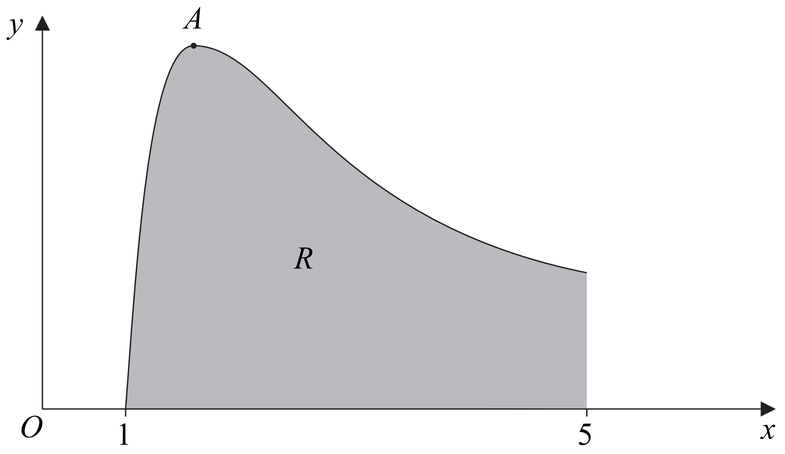
(c) (i) Explain why the function f does **not** have an inverse. *(1 mark)*

(ii) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

9 (a) Given that $y = x^{-2} \ln x$, show that $\dfrac{dy}{dx} = \dfrac{1 - 2\ln x}{x^3}$. *(4 marks)*

(b) Using integration by parts, find $\int x^{-2} \ln x \, dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2} \ln x$.



(i) Using the answer to part (a), find, in terms of e, the x-coordinate of the stationary point A. *(2 marks)*

(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

$$\frac{1}{5}(4 - \ln 5)$$ *(3 marks)*

END OF QUESTIONS

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Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
January 2006
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Insert

Wednesday 25 January 2006 9.00 am to 10.30 am

Insert for use in **Question 6**.

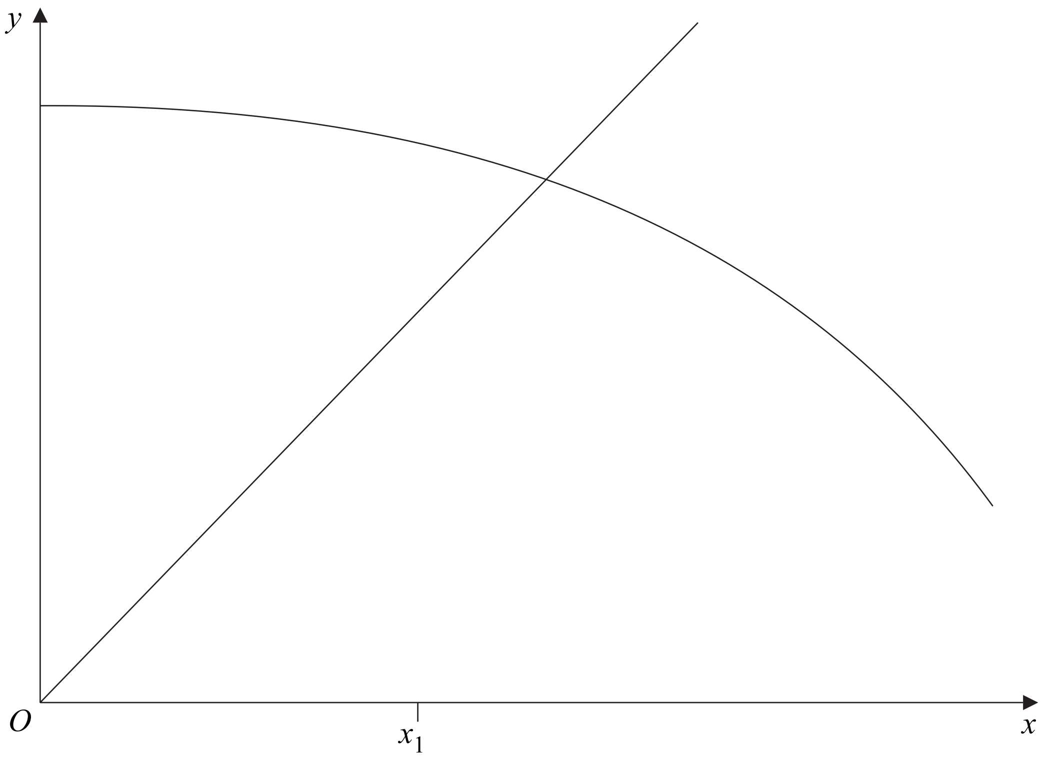
Fill in the boxes at the top of this page.

Attach this insert securely to your answer book.

Turn over for Figure 1

P81600/Jan06/MPC3 6/6/6/

Figure 1 (for Question 6)



General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Thursday 15 June 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The curve $y = x^3 - x - 7$ intersects the x-axis at the point where $x = \alpha$.

(a) Show that α lies between 2.0 and 2.1. *(2 marks)*

(b) Show that the equation $x^3 - x - 7 = 0$ can be rearranged in the form $x = \sqrt[3]{x + 7}$. *(1 mark)*

(c) Use the iteration $x_{n+1} = \sqrt[3]{x_n + 7}$ with $x_1 = 2$ to find the values of x_2, x_3 and x_4, giving your answers to three significant figures. *(3 marks)*

2 (a) Find $\dfrac{dy}{dx}$ when $y = (3x - 1)^{10}$. *(2 marks)*

(b) Use the substitution $u = 2x + 1$ to find $\int x(2x + 1)^8 \, dx$, giving your answer in terms of x. *(4 marks)*

3 (a) Solve the equation $\sec x = 5$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation $\tan^2 x = 3 \sec x + 9$ can be written as

$$\sec^2 x - 3 \sec x - 10 = 0$$ *(2 marks)*

(c) Solve the equation $\tan^2 x = 3 \sec x + 9$, giving all the values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

4 (a) Sketch and label on the same set of axes the graphs of:

(i) $y = |x|$; *(1 mark)*

(ii) $y = |2x - 4|$. *(2 marks)*

(b) (i) Solve the equation $|x| = |2x - 4|$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|x| > |2x - 4|$. *(2 marks)*

5 (a) A curve has equation $y = e^{2x} - 10e^{x} + 12x$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find $\frac{d^2y}{dx^2}$. *(1 mark)*

(b) The points P and Q are the stationary points of the curve.

(i) Show that the x-coordinates of P and Q are given by the solutions of the equation

$$e^{2x} - 5e^{x} + 6 = 0$$ *(1 mark)*

(ii) By using the substitution $z = e^{x}$, or otherwise, show that the x-coordinates of P and Q are $\ln 2$ and $\ln 3$. *(3 marks)*

(iii) Find the y-coordinates of P and Q, giving each of your answers in the form $m + 12\ln n$, where m and n are integers. *(3 marks)*

(iv) Using the answer to part (a)(ii), determine the nature of each stationary point. *(3 marks)*

6 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_1^5 \ln x \, dx$, giving your answer to three significant figures. *(3 marks)*

(b) (i) Given that $y = x\ln x$, find $\frac{dy}{dx}$. *(2 marks)*

(ii) Hence, or otherwise, find $\int \ln x \, dx$. *(2 marks)*

(iii) Find the exact value of $\int_1^5 \ln x \, dx$. *(2 marks)*

7 (a) Given that $z = \frac{\sin x}{\cos x}$, use the quotient rule to show that $\frac{dz}{dx} = \sec^2 x$. *(3 marks)*

(b) Sketch the curve with equation $y = \sec x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. *(2 marks)*

(c) The region R is bounded by the curve $y = \sec x$, the x-axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(3 marks)*

Turn over ▶

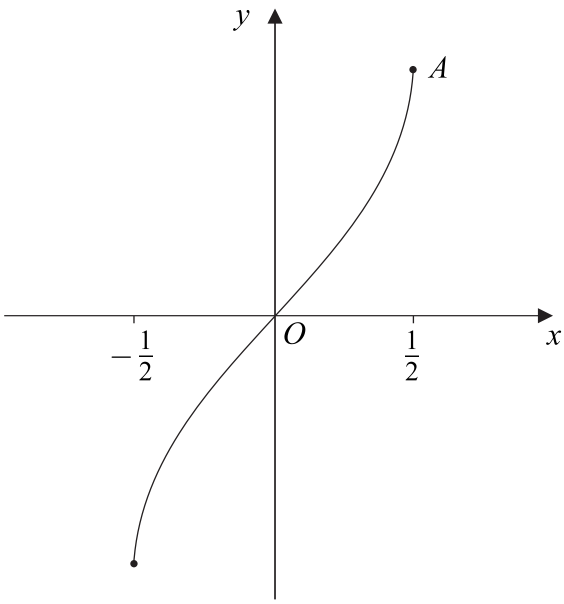
8 A function f is defined by $f(x) = 2e^{3x} - 1$ for all real values of x.

(a) Find the range of f. *(2 marks)*

(b) Show that $f^{-1}(x) = \frac{1}{3}\ln\left(\frac{x+1}{2}\right)$. *(3 marks)*

(c) Find the gradient of the curve $y = f^{-1}(x)$ when $x = 0$. *(4 marks)*

9 The diagram shows the curve with equation $y = \sin^{-1} 2x$, where $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$.



(a) Find the y-coordinate of the point A, where $x = \frac{1}{2}$. *(1 mark)*

(b) (i) Given that $y = \sin^{-1} 2x$, show that $x = \frac{1}{2}\sin y$. *(1 mark)*

(ii) Given that $x = \frac{1}{2}\sin y$, find $\frac{dx}{dy}$ in terms of y. *(1 mark)*

(c) Using the answers to part (b) and a suitable trigonometrical identity, show that

$$\frac{dy}{dx} = \frac{2}{\sqrt{1 - 4x^2}}$$ *(4 marks)*

END OF QUESTIONS

General Certificate of Education
January 2007
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Thursday 18 January 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Use the mid-ordinate rule with four strips of equal width to find an estimate for $\int_1^5 \frac{1}{1+\ln x}\,dx$, giving your answer to three significant figures. *(4 marks)*

2 Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sec x$ onto the graph of $y = 1 + \sec 3x$. *(4 marks)*

3 The functions f and g are defined with their respective domains by

$$f(x) = 3 - x^2, \quad \text{for all real values of } x$$

$$g(x) = \frac{2}{x+1}, \quad \text{for real values of } x,\ x \neq -1$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of g is g^{-1}.

(i) Find $g^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) The composite function gf is denoted by h.

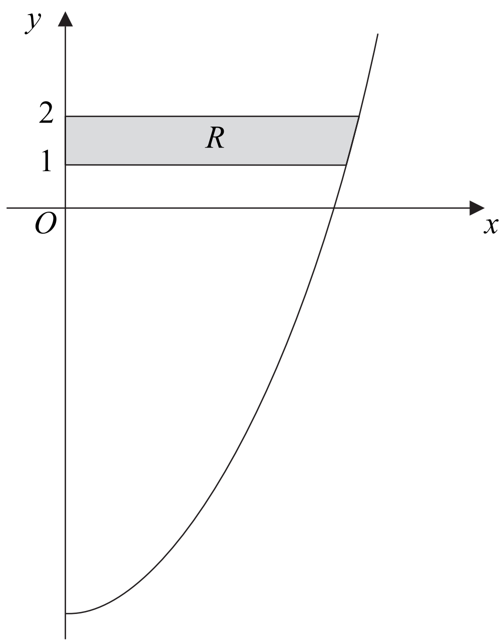
(i) Find $h(x)$, simplifying your answer. *(2 marks)*

(ii) State the greatest possible domain of h. *(1 mark)*

4 (a) Use integration by parts to find $\int x \sin x \, dx$. *(4 marks)*

(b) Using the substitution $u = x^2 + 5$, or otherwise, find $\int x\sqrt{x^2+5} \, dx$. *(4 marks)*

(c) The diagram shows the curve $y = x^2 - 9$ for $x \geqslant 0$.



The shaded region R is bounded by the curve, the lines $y = 1$ and $y = 2$, and the y-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° **about the *y*-axis**. *(4 marks)*

5 (a) (i) Show that the equation

$$2\cot^2 x + 5\,\mathrm{cosec}\, x = 10$$

can be written in the form $2\,\mathrm{cosec}^2 x + 5\,\mathrm{cosec}\, x - 12 = 0$. *(2 marks)*

(ii) Hence show that $\sin x = -\frac{1}{4}$ or $\sin x = \frac{2}{3}$. *(3 marks)*

(b) Hence, or otherwise, solve the equation

$$2\cot^2(\theta - 0.1) + 5\,\mathrm{cosec}(\theta - 0.1) = 10$$

giving all values of θ in radians to two decimal places in the interval $-\pi < \theta < \pi$. *(3 marks)*

Turn over ▶

6 (a) Find $\dfrac{dy}{dx}$ when:

(i) $y = (4x^2 + 3x + 2)^{10}$; *(2 marks)*

(ii) $y = x^2 \tan x$. *(2 marks)*

(b) (i) Find $\dfrac{dx}{dy}$ when $x = 2y^3 + \ln y$. *(1 mark)*

(ii) Hence find an equation of the tangent to the curve $x = 2y^3 + \ln y$ at the point $(2,1)$. *(3 marks)*

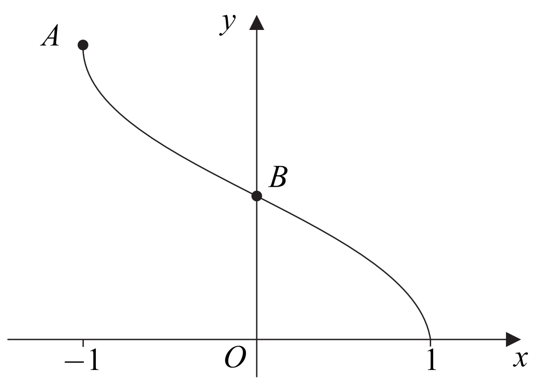
7 (a) Sketch the graph of $y = |2x|$. *(1 mark)*

(b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(3 marks)*

(c) Solve $4 - |2x| = x$. *(3 marks)*

(d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. *(2 marks)*

8 The diagram shows the curve $y = \cos^{-1} x$ for $-1 \leqslant x \leqslant 1$.

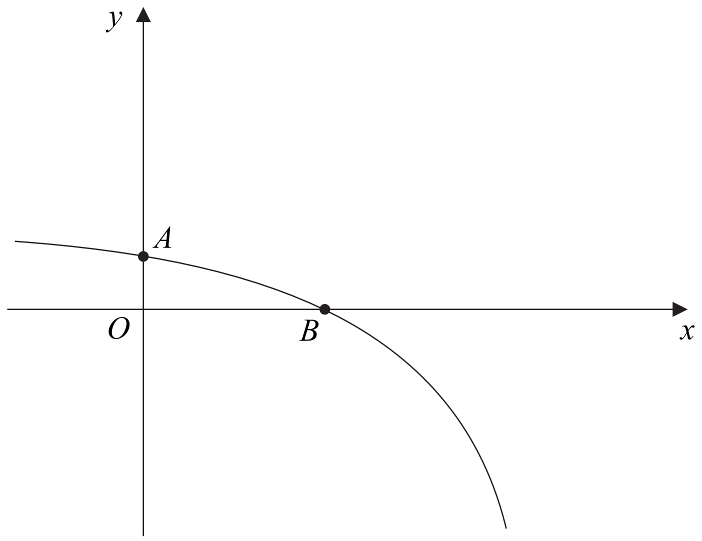


(a) Write down the exact coordinates of the points A and B. *(2 marks)*

(b) The equation $\cos^{-1} x = 3x + 1$ has only one root. Given that the root of this equation is α, show that $0.1 \leqslant \alpha \leqslant 0.2$. *(2 marks)*

(c) Use the iteration $x_{n+1} = \frac{1}{3}(\cos^{-1} x_n - 1)$ with $x_1 = 0.1$ to find the values of x_2, x_3 and x_4, giving your answers to three decimal places. *(3 marks)*

9 The sketch shows the graph of $y = 4 - e^{2x}$. The curve crosses the y-axis at the point A and the x-axis at the point B.



(a) (i) Find $\int (4 - e^{2x})\,dx$. *(2 marks)*

(ii) Hence show that $\int_0^{\ln 2} (4 - e^{2x})\,dx = 4\ln 2 - \frac{3}{2}$. *(2 marks)*

(b) (i) Write down the y-coordinate of A. *(1 mark)*

(ii) Show that $x = \ln 2$ at B. *(2 marks)*

(c) Find the equation of the normal to the curve $y = 4 - e^{2x}$ at the point B. *(4 marks)*

(d) Find the area of the region enclosed by the curve $y = 4 - e^{2x}$, the normal to the curve at B and the y-axis. *(3 marks)*

END OF QUESTIONS

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General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Monday 11 June 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Question 4 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

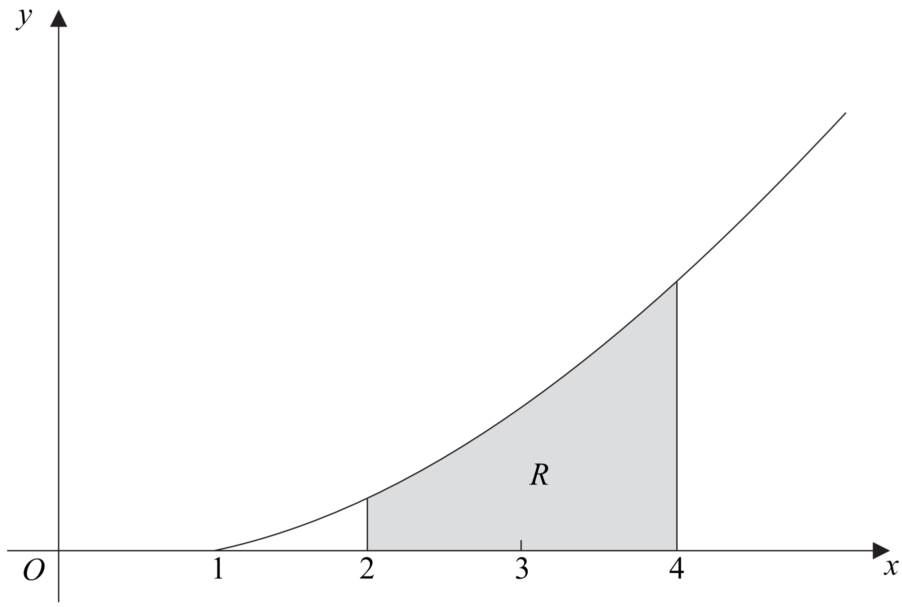
1 (a) Differentiate $\ln x$ with respect to x. *(1 mark)*

(b) Given that $y = (x+1)\ln x$, find $\dfrac{dy}{dx}$. *(2 marks)*

(c) Find an equation of the normal to the curve $y = (x+1)\ln x$ at the point where $x = 1$. *(4 marks)*

2 (a) Differentiate $(x-1)^4$ with respect to x. *(1 mark)*

(b) The diagram shows the curve with equation $y = 2\sqrt{(x-1)^3}$ for $x \geqslant 1$.



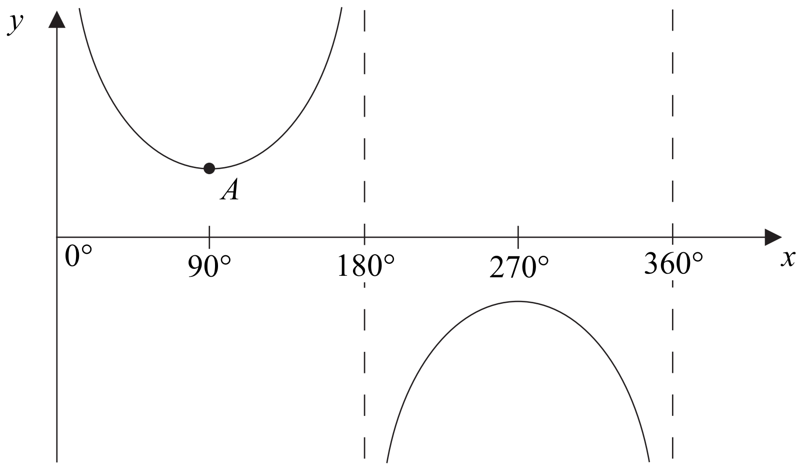
The shaded region R is bounded by the curve $y = 2\sqrt{(x-1)^3}$, the lines $x = 2$ and $x = 4$, and the x-axis.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

(c) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sqrt{x^3}$ onto the graph of $y = 2\sqrt{(x-1)^3}$. *(4 marks)*

3 (a) Solve the equation $\operatorname{cosec} x = 2$, giving all values of x in the interval $0° < x < 360°$. *(2 marks)*

(b) The diagram shows the graph of $y = \operatorname{cosec} x$ for $0° < x < 360°$.



(i) The point A on the curve is where $x = 90°$. State the y-coordinate of A. *(1 mark)*

(ii) Sketch the graph of $y = |\operatorname{cosec} x|$ for $0° < x < 360°$. *(2 marks)*

(c) Solve the equation $|\operatorname{cosec} x| = 2$, giving all values of x in the interval $0° < x < 360°$. *(2 marks)*

Turn over for the next question

4 [**Figure 1**, printed on the insert, is provided for use in this question.]

(a) Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to $\int_1^2 3^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(b) The curve $y = 3^x$ intersects the line $y = x + 3$ at the point where $x = \alpha$.

(i) Show that α lies between 0.5 and 1.5. *(2 marks)*

(ii) Show that the equation $3^x = x + 3$ can be rearranged into the form

$$x = \frac{\ln(x+3)}{\ln 3}$$ *(2 marks)*

(iii) Use the iteration $x_{n+1} = \dfrac{\ln(x_n + 3)}{\ln 3}$ with $x_1 = 0.5$ to find x_3 to two significant figures. *(2 marks)*

(iv) The sketch on **Figure 1** shows part of the graphs of $y = \dfrac{\ln(x+3)}{\ln 3}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

5 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{x-2} \quad \text{for } x \geqslant 2$$

$$g(x) = \frac{1}{x} \quad \text{for real values of } x, \ x \neq 0$$

(a) State the range of f. *(2 marks)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 1$. *(3 marks)*

(c) The inverse of f is f^{-1}. Find $f^{-1}(x)$. *(3 marks)*

6 (a) Use integration by parts to find $\int x\mathrm{e}^{5x}\,\mathrm{d}x$. *(4 marks)*

(b) (i) Use the substitution $u = \sqrt{x}$ to show that

$$\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,\mathrm{d}x = \int \frac{2}{1+u}\,\mathrm{d}u$$ *(2 marks)*

(ii) Find the exact value of $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})}\,\mathrm{d}x$. *(3 marks)*

7 (a) A curve has equation $y = (x^2 - 3)\mathrm{e}^x$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(ii) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(2 marks)*

(b) (i) Find the x-coordinate of each of the stationary points of the curve. *(4 marks)*

(ii) Using your answer to part (a)(ii), determine the nature of each of the stationary points. *(2 marks)*

8 (a) Write down $\int \sec^2 x\,\mathrm{d}x$. *(1 mark)*

(b) Given that $y = \frac{\cos x}{\sin x}$, use the quotient rule to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosec}^2 x$. *(4 marks)*

(c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \operatorname{cosec}^2 x$. *(3 marks)*

(d) Hence find $\int_{0.5}^{1} (\tan x + \cot x)^2\,\mathrm{d}x$, giving your answer to two significant figures. *(4 marks)*

END OF QUESTIONS

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Surname		Other Names								
Centre Number						Candidate Number				
Candidate Signature										

General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Insert

Insert for use in **Question 4**.

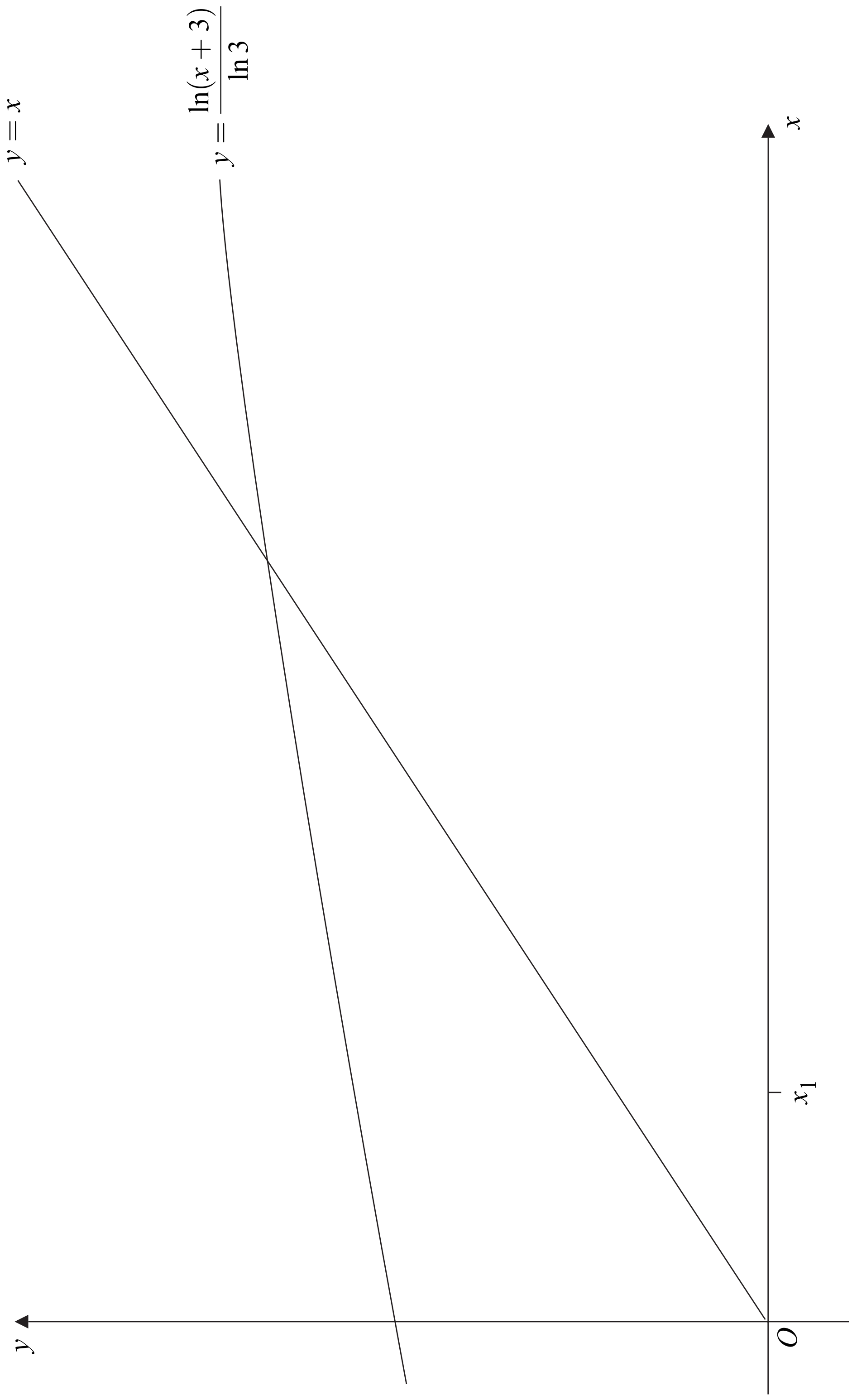
Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 4)



General Certificate of Education
January 2008
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Thursday 17 January 2008 1.30 pm to 3.00 pm

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- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Find $\frac{dy}{dx}$ when:

(i) $y = (2x^2 - 5x + 1)^{20}$; *(2 marks)*

(ii) $y = x \cos x$. *(2 marks)*

(b) Given that

$$y = \frac{x^3}{x-2}$$

show that

$$\frac{dy}{dx} = \frac{kx^2(x-3)}{(x-2)^2}$$

where k is a positive integer. *(3 marks)*

2 (a) Solve the equation $\cot x = 2$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(2 marks)*

(b) Show that the equation $\operatorname{cosec}^2 x = \frac{3\cot x + 4}{2}$ can be written as

$$2\cot^2 x - 3\cot x - 2 = 0$$ *(2 marks)*

(c) Solve the equation $\operatorname{cosec}^2 x = \frac{3\cot x + 4}{2}$, giving all values of x in the interval $0 \leqslant x \leqslant 2\pi$ in radians to two decimal places. *(4 marks)*

3 The equation

$$x + (1 + 3x)^{\frac{1}{4}} = 0$$

has a single root, α.

(a) Show that α lies between -0.33 and -0.32. *(2 marks)*

(b) Show that the equation $x + (1 + 3x)^{\frac{1}{4}} = 0$ can be rearranged into the form

$$x = \tfrac{1}{3}(x^4 - 1)$$ *(2 marks)*

(c) Use the iteration $x_{n+1} = \dfrac{(x_n{}^4 - 1)}{3}$ with $x_1 = -0.3$ to find x_4, giving your answer to three significant figures. *(3 marks)*

4 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^3, \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x-3}, \qquad \text{for real values of } x,\ x \neq 3$$

(a) State the range of f. *(1 mark)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 64$. *(3 marks)*

(c) (i) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

5 (a) (i) Given that $y = 2x^2 - 8x + 3$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(1 mark)*

(ii) Hence, or otherwise, find

$$\int_4^6 \frac{x-2}{2x^2 - 8x + 3}\,\mathrm{d}x$$

giving your answer in the form $k \ln 3$, where k is a rational number. *(4 marks)*

(b) Use the substitution $u = 3x - 1$ to find $\int x\sqrt{3x-1}\,\mathrm{d}x$, giving your answer in terms of x. *(4 marks)*

Turn over for the next question

Turn over ▶

6 (a) Sketch the curve with equation $y = \operatorname{cosec} x$ for $0 < x < \pi$. *(2 marks)*

(b) Use the mid-ordinate rule with four strips to find an estimate for $\int_{0.1}^{0.5} \operatorname{cosec} x \, dx$, giving your answer to three significant figures. *(4 marks)*

7 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 4x^2 - 5$. *(4 marks)*

(b) Sketch the graph of $y = |4x^2 - 5|$, indicating the coordinates of the point where the curve crosses the y-axis. *(3 marks)*

(c) (i) Solve the equation $|4x^2 - 5| = 4$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|4x^2 - 5| \geqslant 4$. *(2 marks)*

8 (a) Given that $e^{-2x} = 3$, find the exact value of x. *(2 marks)*

(b) Use integration by parts to find $\int x e^{-2x} \, dx$. *(4 marks)*

(c) A curve has equation $y = e^{-2x} + 6x$.

(i) Find the exact values of the coordinates of the stationary point of the curve. *(4 marks)*

(ii) Determine the nature of the stationary point. *(2 marks)*

(iii) The region R is bounded by the curve $y = e^{-2x} + 6x$, the x-axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(5 marks)*

END OF QUESTIONS

General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Friday 23 May 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Find $\frac{dy}{dx}$ when:

(a) $y = (3x+1)^5$; *(2 marks)*

(b) $y = \ln(3x+1)$; *(2 marks)*

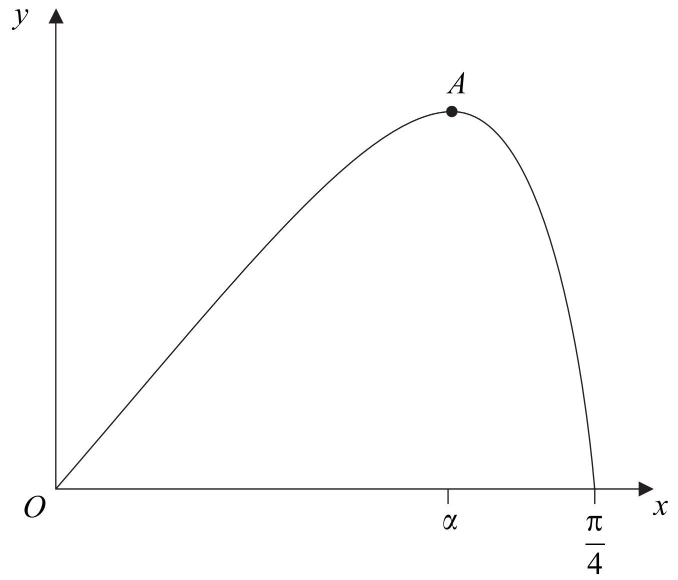
(c) $y = (3x+1)^5 \ln(3x+1)$. *(3 marks)*

2 (a) Solve the equation $\sec x = 3$, giving the values of x in radians to two decimal places in the interval $0 \leqslant x < 2\pi$. *(3 marks)*

(b) Show that the equation $\tan^2 x = 2\sec x + 2$ can be written as $\sec^2 x - 2\sec x - 3 = 0$. *(2 marks)*

(c) Solve the equation $\tan^2 x = 2\sec x + 2$, giving the values of x in radians to two decimal places in the interval $0 \leqslant x < 2\pi$. *(4 marks)*

3 A curve is defined for $0 \leqslant x \leqslant \frac{\pi}{4}$ by the equation $y = x\cos 2x$, and is sketched below.



(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(b) The point A, where $x = \alpha$, on the curve is a stationary point.

(i) Show that $1 - 2\alpha\tan 2\alpha = 0$. *(2 marks)*

(ii) Show that $0.4 < \alpha < 0.5$. *(2 marks)*

(iii) Show that the equation $1 - 2x\tan 2x = 0$ can be rearranged to become $x = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x}\right)$. *(1 mark)*

(iv) Use the iteration $x_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x_n}\right)$ with $x_1 = 0.4$ to find x_3, giving your answer to two significant figures. *(2 marks)*

(c) Use integration by parts to find $\int_0^{0.5} x\cos 2x \, \mathrm{d}x$, giving your answer to three significant figures. *(5 marks)*

Turn over ▶

4 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^2, \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{2x-3}, \qquad \text{for real values of } x,\ x \neq \frac{3}{2}$$

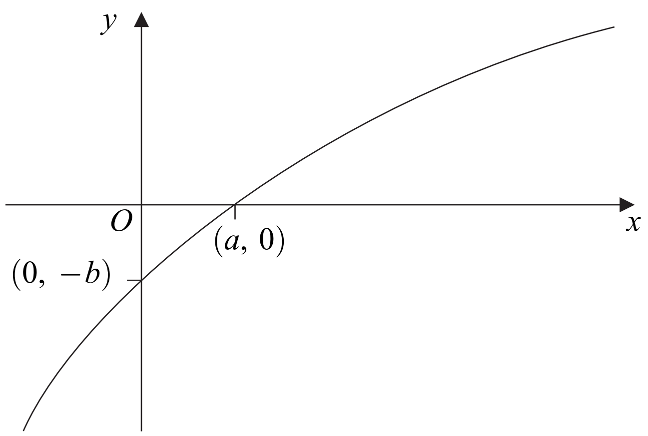
(a) State the range of f. *(1 mark)*

(b) (i) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) Solve the equation $\mathrm{fg}(x) = 9$. *(3 marks)*

5 (a) The diagram shows part of the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at the point $(a, 0)$ and the y-axis at the point $(0, -b)$.



On separate diagrams, sketch the curves with the following equations. On each diagram, indicate, in terms of a or b, the coordinates of the points where the curve crosses the coordinate axes.

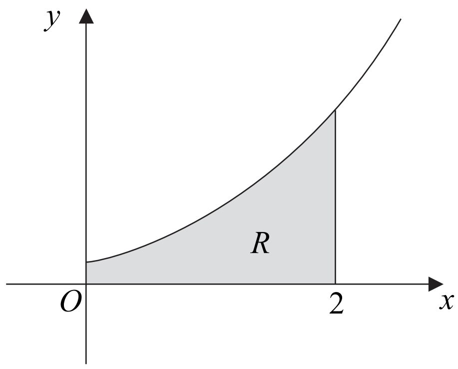
(i) $y = |\,\mathrm{f}(x)\,|$. *(2 marks)*

(ii) $y = 2\mathrm{f}(x)$. *(2 marks)*

(b) (i) Describe a sequence of geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 4\ln(x+1) - 2$. *(6 marks)*

(ii) Find the exact values of the coordinates of the points where the graph of $y = 4\ln(x+1) - 2$ crosses the coordinate axes. *(4 marks)*

6 The diagram shows the curve with equation $y = (e^{3x} + 1)^{\frac{1}{2}}$ for $x \geqslant 0$.



(a) Find the gradient of the curve $y = (e^{3x} + 1)^{\frac{1}{2}}$ at the point where $x = \ln 2$. *(5 marks)*

(b) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^2 (e^{3x} + 1)^{\frac{1}{2}} \, dx$, giving your answer to three significant figures. *(4 marks)*

(c) The shaded region R is bounded by the curve, the lines $x = 0$, $x = 2$ and the x-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° about the x-axis. *(4 marks)*

7 (a) Given that $y = \dfrac{\sin\theta}{\cos\theta}$, use the quotient rule to show that $\dfrac{dy}{d\theta} = \sec^2\theta$. *(3 marks)*

(b) Given that $x = \sin\theta$, show that $\dfrac{x}{\sqrt{1 - x^2}} = \tan\theta$. *(2 marks)*

(c) Use the substitution $x = \sin\theta$ to find $\displaystyle\int \frac{1}{(1 - x^2)^{\frac{3}{2}}} \, dx$, giving your answer in terms of x. *(5 marks)*

END OF QUESTIONS

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General Certificate of Education
January 2009
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Monday 19 January 2009 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 3 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

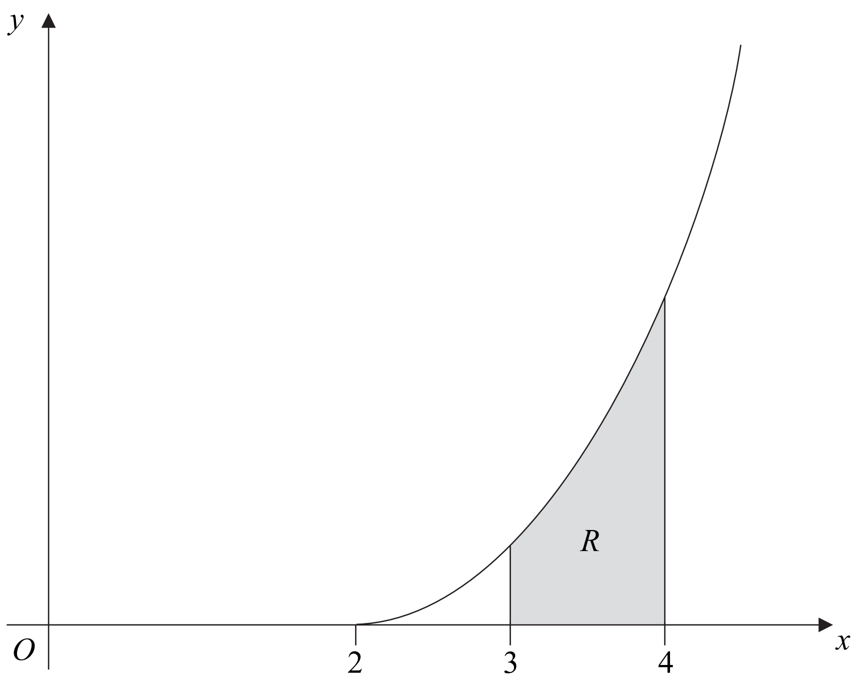
Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to $\int_1^9 \frac{1}{1+\sqrt{x}}\,dx$, giving your answer to three significant figures. *(4 marks)*

2 The diagram shows the curve with equation $y = \sqrt{(x-2)^5}$ for $x \geqslant 2$.



The shaded region R is bounded by the curve $y = \sqrt{(x-2)^5}$, the x-axis and the lines $x = 3$ and $x = 4$.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve with equation $y = x^3 + 5x - 4$ intersects the x-axis at the point A, where $x = \alpha$.

(a) Show that α lies between 0.5 and 1. *(2 marks)*

(b) Show that the equation $x^3 + 5x - 4 = 0$ can be rearranged into the form

$$x = \frac{1}{5}(4 - x^3)$$ *(1 mark)*

(c) Use the iteration $x_{n+1} = \frac{1}{5}(4 - x_n{}^3)$ with $x_1 = 0.5$ to find x_3, giving your answer to three decimal places. *(2 marks)*

(d) The sketch on **Figure 1** shows parts of the graphs of $y = \frac{1}{5}(4 - x^3)$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

4 (a) Solve the equation $\sec x = \frac{3}{2}$, giving all values of x to the nearest degree in the interval $0° < x < 360°$. *(2 marks)*

(b) By using a suitable trigonometrical identity, solve the equation

$$2\tan^2 x = 10 - 5\sec x$$

giving all values of x to the nearest degree in the interval $0° < x < 360°$. *(6 marks)*

Turn over for the next question

Turn over ▶

5 The functions f and g are defined with their respective domains by

$$f(x) = 2 - x^4 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{x-4} \qquad \text{for real values of } x,\ x \neq 4$$

(a) State the range of f. *(2 marks)*

(b) Explain why the function f does not have an inverse. *(1 mark)*

(c) (i) Write down an expression for $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = -14$. *(3 marks)*

6 A curve has equation $y = e^{2x}(x^2 - 4x - 2)$.

(a) Find the value of the x-coordinate of each of the stationary points of the curve. *(6 marks)*

(b) (i) Find $\dfrac{d^2y}{dx^2}$. *(2 marks)*

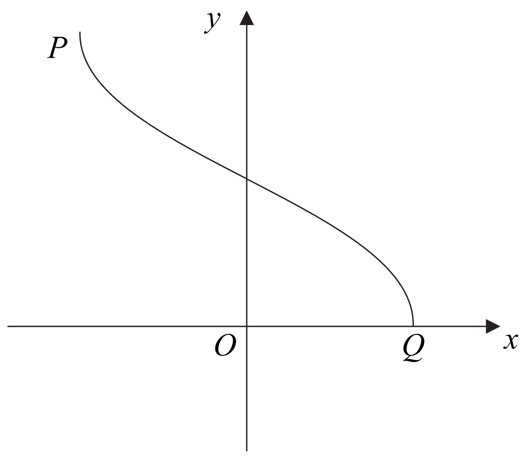
(ii) Determine the nature of each of the stationary points of the curve. *(2 marks)*

7 (a) Given that $3e^x = 4$, find the exact value of x. *(2 marks)*

(b) (i) By substituting $y = e^x$, show that the equation $3e^x + 20e^{-x} = 19$ can be written as $3y^2 - 19y + 20 = 0$. *(1 mark)*

(ii) Hence solve the equation $3e^x + 20e^{-x} = 19$, giving your answers as exact values. *(3 marks)*

8 The sketch shows the graph of $y = \cos^{-1} x$.



(a) Write down the coordinates of P and Q, the end points of the graph. *(2 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \cos^{-1} x$ onto the graph of $y = 2\cos^{-1}(x - 1)$. *(4 marks)*

(c) Sketch the graph of $y = 2\cos^{-1}(x - 1)$. *(2 marks)*

(d) (i) Write the equation $y = 2\cos^{-1}(x - 1)$ in the form $x = \mathrm{f}(y)$. *(2 marks)*

(ii) Hence find the value of $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ when $y = 2$. *(3 marks)*

9 (a) Given that $y = \dfrac{4x}{4x - 3}$, use the quotient rule to show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k}{(4x - 3)^2}$, where k is an integer. *(2 marks)*

(b) (i) Given that $y = x\ln(4x - 3)$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(ii) Find an equation of the tangent to the curve $y = x\ln(4x - 3)$ at the point where $x = 1$. *(3 marks)*

(c) (i) Use the substitution $u = 4x - 3$ to find $\displaystyle\int \frac{4x}{4x - 3}\,\mathrm{d}x$, giving your answer in terms of x. *(4 marks)*

(ii) By using integration by parts, or otherwise, find $\displaystyle\int \ln(4x - 3)\,\mathrm{d}x$. *(4 marks)*

END OF QUESTIONS

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General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Friday 5 June 2009 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) The curve with equation

$$y = \frac{\cos x}{2x+1}, \qquad x > -\frac{1}{2}$$

intersects the line $y = \frac{1}{2}$ at the point where $x = \alpha$.

(i) Show that α lies between 0 and $\frac{\pi}{2}$. *(2 marks)*

(ii) Show that the equation $\frac{\cos x}{2x+1} = \frac{1}{2}$ can be rearranged into the form

$$x = \cos x - \frac{1}{2}$$ *(1 mark)*

(iii) Use the iteration $x_{n+1} = \cos x_n - \frac{1}{2}$ with $x_1 = 0$ to find x_3, giving your answer to three decimal places. *(2 marks)*

(b) (i) Given that $y = \frac{\cos x}{2x+1}$, use the quotient rule to find an expression for $\frac{dy}{dx}$. *(3 marks)*

(ii) Hence find the gradient of the normal to the curve $y = \frac{\cos x}{2x+1}$ at the point on the curve where $x = 0$. *(2 marks)*

2 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \sqrt{2x+5}, \qquad \text{for real values of } x, \ x \geqslant -2.5$$

$$\mathrm{g}(x) = \frac{1}{4x+1}, \qquad \text{for real values of } x, \ x \neq -0.25$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) State the domain of f^{-1}. *(1 mark)*

(c) The composite function fg is denoted by h.

(i) Find an expression for $\mathrm{h}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{h}(x) = 3$. *(3 marks)*

3 (a) Solve the equation $\tan x = -\frac{1}{3}$, giving all the values of x in the interval $0 < x < 2\pi$ in radians to two decimal places. *(3 marks)*

(b) Show that the equation

$$3\sec^2 x = 5(\tan x + 1)$$

can be written in the form $3\tan^2 x - 5\tan x - 2 = 0$. *(1 mark)*

(c) Hence, or otherwise, solve the equation

$$3\sec^2 x = 5(\tan x + 1)$$

giving all the values of x in the interval $0 < x < 2\pi$ in radians to two decimal places. *(4 marks)*

Turn over ▶

4 (a) Sketch the graph of $y = |50 - x^2|$, indicating the coordinates of the point where the graph crosses the y-axis. *(3 marks)*

(b) Solve the equation $|50 - x^2| = 14$. *(3 marks)*

(c) Hence, or otherwise, solve the inequality $|50 - x^2| > 14$. *(2 marks)*

(d) Describe a sequence of two geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 50 - x^2$. *(4 marks)*

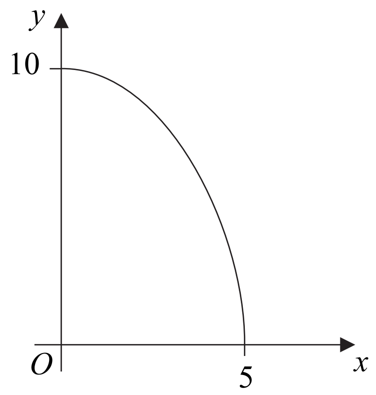
5 (a) Given that $2\ln x = 5$, find the exact value of x. *(1 mark)*

(b) Solve the equation

$$2\ln x + \frac{15}{\ln x} = 11$$

giving your answers as exact values of x. *(5 marks)*

6 The diagram shows the curve with equation $y = \sqrt{100 - 4x^2}$, where $x \geqslant 0$.



(a) Calculate the volume of the solid generated when the region bounded by the curve shown above and the coordinate axes is rotated through 360° about the ***y*-axis**, giving your answer in terms of π. *(5 marks)*

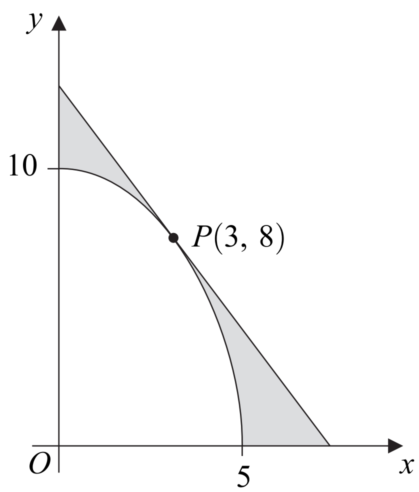
(b) Use the mid-ordinate rule with five strips of equal width to find an estimate for $\int_0^5 \sqrt{100 - 4x^2}\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

(c) The point P on the curve has coordinates (3, 8).

(i) Find the gradient of the curve $y = \sqrt{100 - 4x^2}$ at the point P. *(3 marks)*

(ii) Hence show that the equation of the tangent to the curve at the point P can be written as $2y + 3x = 25$. *(2 marks)*

(d) The shaded regions on the diagram below are bounded by the curve, the tangent at P and the coordinate axes.



Use your answers to part (b) and part (c)(ii) to find an approximate value for the **total** area of the shaded regions. Give your answer to three significant figures. *(5 marks)*

7 (a) Use integration by parts to find $\int (t-1)\ln t \, dt$. *(4 marks)*

(b) Use the substitution $t = 2x + 1$ to show that $\int 4x \ln(2x+1) \, dx$ can be written as $\int (t-1)\ln t \, dt$. *(3 marks)*

(c) Hence find the exact value of $\int_0^1 4x \ln(2x+1) \, dx$. *(3 marks)*

END OF QUESTIONS

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General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MPC3

Unit Pure Core 3

Friday 15 January 2010 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 2 (enclosed).

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A curve has equation $y = \mathrm{e}^{-4x}(x^2 + 2x - 2)$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\mathrm{e}^{-4x}(5 - 3x - 2x^2)$. *(3 marks)*

(b) Find the exact values of the coordinates of the stationary points of the curve. *(5 marks)*

2 [**Figure 1**, printed on the insert, is provided for use in this question.]

(a) (i) Sketch the graph of $y = \sin^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. *(3 marks)*

(ii) By drawing a suitable straight line on your sketch, show that the equation

$$\sin^{-1} x = \tfrac{1}{4}x + 1$$

has only one solution. *(2 marks)*

(b) The root of the equation $\sin^{-1} x = \frac{1}{4}x + 1$ is α. Show that $0.5 < \alpha < 1$. *(2 marks)*

(c) The equation $\sin^{-1} x = \frac{1}{4}x + 1$ can be rewritten as $x = \sin\left(\frac{1}{4}x + 1\right)$.

(i) Use the iteration $x_{n+1} = \sin\left(\frac{1}{4}x_n + 1\right)$ with $x_1 = 0.5$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \sin\left(\frac{1}{4}x + 1\right)$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

3 (a) Solve the equation

$$\operatorname{cosec} x = 3$$

giving all values of x in radians to two decimal places, in the interval $0 \leqslant x \leqslant 2\pi$. *(2 marks)*

(b) By using a suitable trigonometric identity, solve the equation

$$\cot^2 x = 11 - \operatorname{cosec} x$$

giving all values of x in radians to two decimal places, in the interval $0 \leqslant x \leqslant 2\pi$. *(6 marks)*

4 (a) Sketch the graph of $y = |8 - 2x|$. *(2 marks)*

(b) Solve the equation $|8 - 2x| = 4$. *(2 marks)*

(c) Solve the inequality $|8 - 2x| > 4$. *(2 marks)*

5 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^{12} \ln(x^2 + 5)\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

(b) A curve has equation $y = \ln(x^2 + 5)$.

(i) Show that this equation can be rewritten as $x^2 = \mathrm{e}^y - 5$. *(1 mark)*

(ii) The region bounded by the curve, the lines $y = 5$ and $y = 10$ and the y-axis is rotated through 360° about the y-axis. Find the exact value of the volume of the solid generated. *(4 marks)*

(c) The graph with equation $y = \ln(x^2 + 5)$ is stretched with scale factor 4 parallel to the x-axis, and then translated through $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ to give the graph with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(3 marks)*

Turn over for the next question

6 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \mathrm{e}^{2x} - 3, \quad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{3x+4}, \quad \text{for real values of } x,\ x \neq -\frac{4}{3}$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 0$. *(2 marks)*

(c) (i) Find an expression for $\mathrm{gf}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{gf}(x) = 1$, giving your answer in an exact form. *(3 marks)*

7 It is given that $y = \tan 4x$.

(a) By writing $\tan 4x$ as $\dfrac{\sin 4x}{\cos 4x}$, use the quotient rule to show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = p(1 + \tan^2 4x)$, where p is a number to be determined. *(3 marks)*

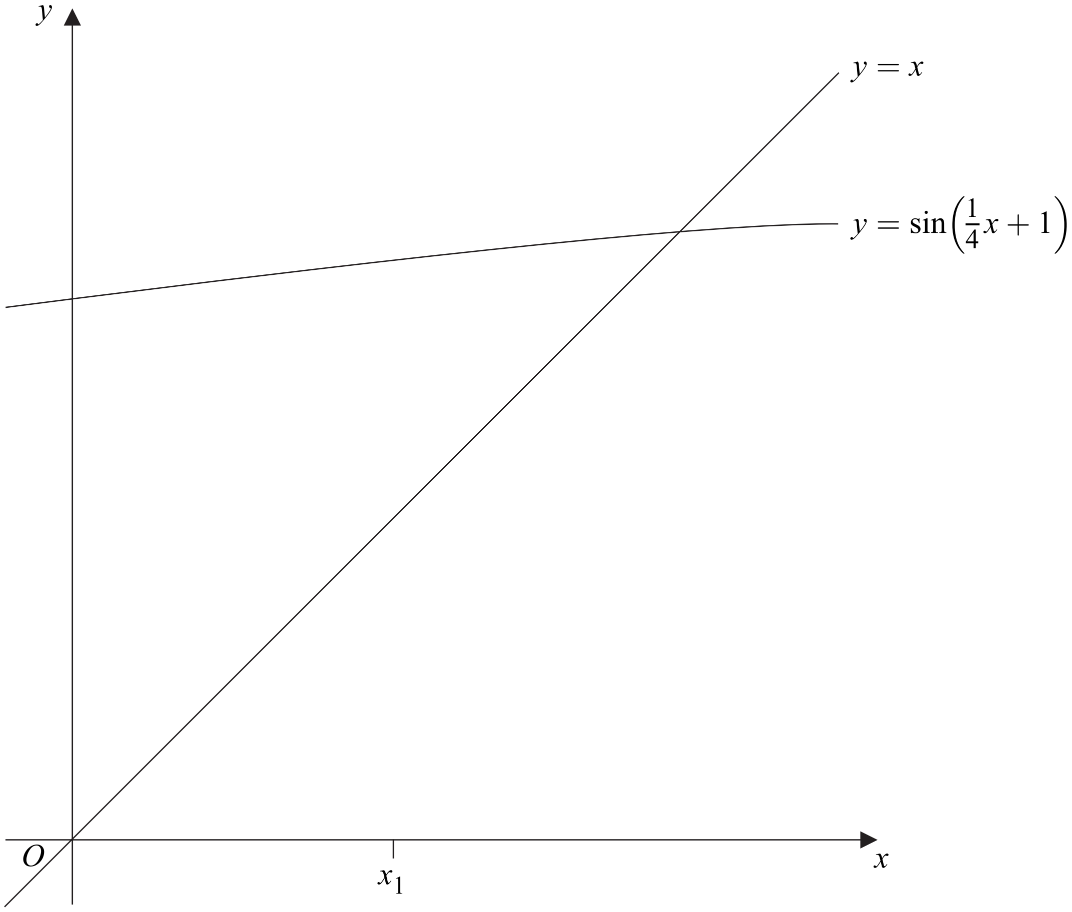
(b) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = qy(1 + y^2)$, where q is a number to be determined. *(5 marks)*

8 (a) Using integration by parts, find $\displaystyle\int x \sin(2x - 1)\,\mathrm{d}x$. *(5 marks)*

(b) Use the substitution $u = 2x - 1$ to find $\displaystyle\int \frac{x^2}{2x-1}\,\mathrm{d}x$, giving your answer in terms of x. *(6 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 2)



General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MPC3

Unit Pure Core 3

Friday 11 June 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The curve $y = 3^x$ intersects the curve $y = 10 - x^3$ at the point where $x = \alpha$.

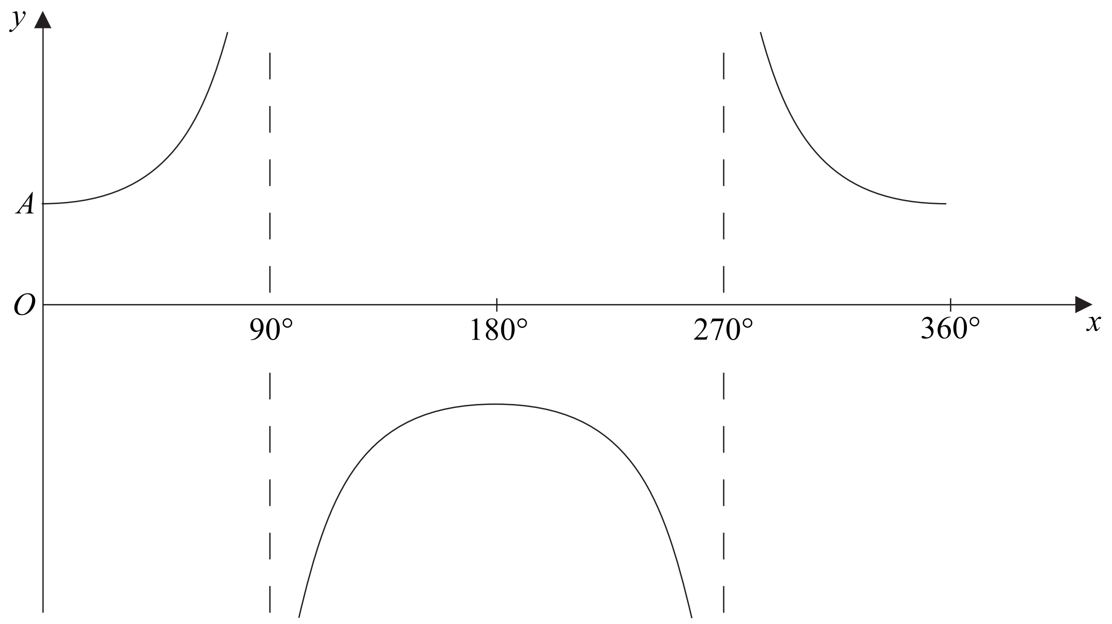
(a) Show that α lies between 1 and 2. *(2 marks)*

(b) (i) Show that the equation $3^x = 10 - x^3$ can be rearranged into the form $x = \sqrt[3]{10 - 3^x}$. *(1 mark)*

(ii) Use the iteration $x_{n+1} = \sqrt[3]{10 - 3^{x_n}}$ with $x_1 = 1$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

---

2 (a) The diagram shows the graph of $y = \sec x$ for $0° \leqslant x \leqslant 360°$.

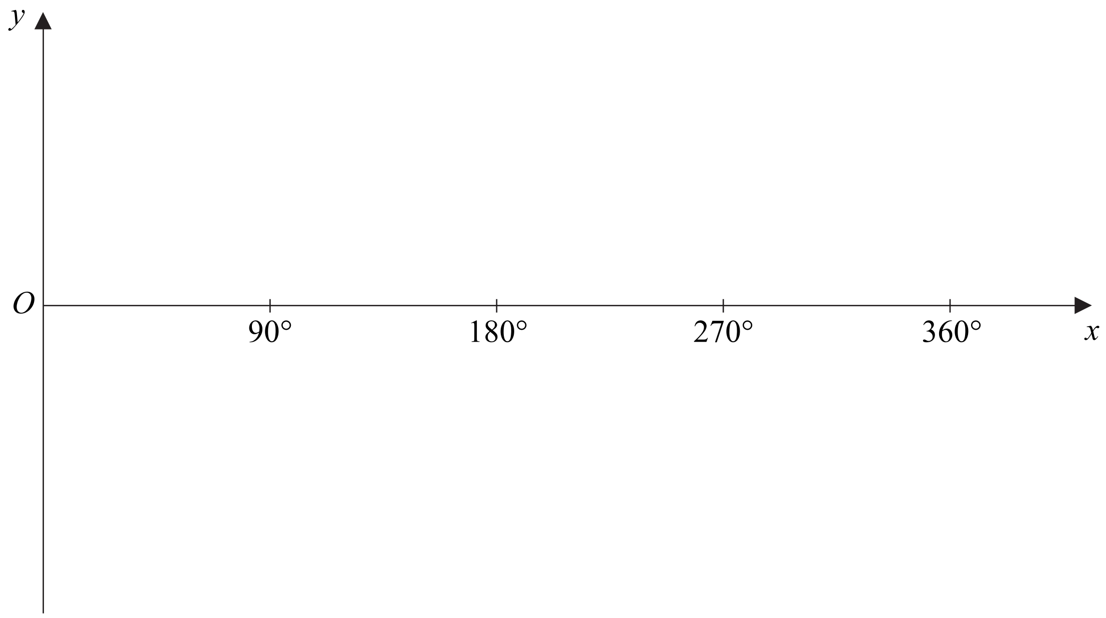


(i) The point A on the curve is where $x = 0$. State the y-coordinate of A. *(1 mark)*

(ii) Sketch, on the axes given on page **3**, the graph of $y = |\sec 2x|$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Solve the equation $\sec x = 2$, giving all values of x in degrees in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) Solve the equation $|\sec(2x - 10°)| = 2$, giving all values of x in degrees in the interval $0° \leqslant x \leqslant 180°$. *(4 marks)*



3 (a) Find $\frac{dy}{dx}$ when:

(i) $y = \ln(5x - 2)$; *(2 marks)*

(ii) $y = \sin 2x$. *(2 marks)*

(b) The functions f and g are defined with their respective domains by

$\mathrm{f}(x) = \ln(5x - 2)$, for real values of x such that $x \geqslant \frac{1}{2}$

$\mathrm{g}(x) = \sin 2x$, for real values of x in the interval $-\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}$

(i) Find the range of f. *(2 marks)*

(ii) Find an expression for $\mathrm{gf}(x)$. *(1 mark)*

(iii) Solve the equation $\mathrm{gf}(x) = 0$. *(3 marks)*

(iv) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(2 marks)*

4 (a) Use Simpson's rule with 7 ordinates (6 strips) to find an approximation to $\displaystyle\int_{0.5}^{2} \frac{x}{1+x^3}\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) Find the exact value of $\displaystyle\int_{0}^{1} \frac{x^2}{1+x^3}\,dx$. *(4 marks)*

---

5 (a) Show that the equation

$$10\operatorname{cosec}^2 x = 16 - 11\cot x$$

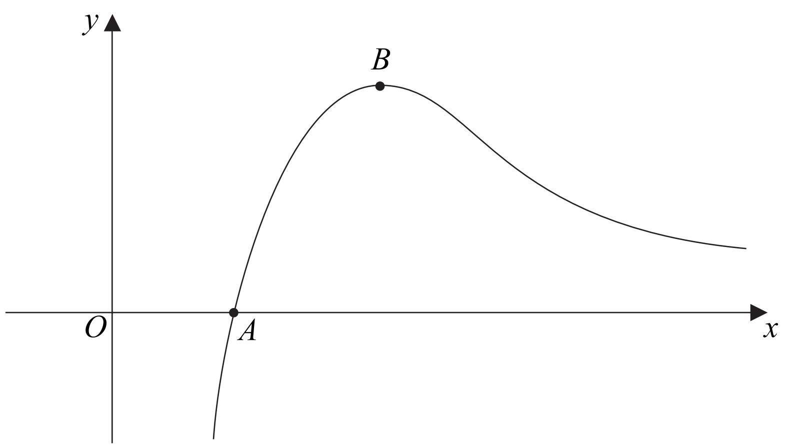
can be written in the form

$$10\cot^2 x + 11\cot x - 6 = 0$$ *(1 mark)*

(b) Hence, given that $10\operatorname{cosec}^2 x = 16 - 11\cot x$, find the possible values of $\tan x$. *(4 marks)*

---

6 The diagram shows the curve $y = \dfrac{\ln x}{x}$.



The curve crosses the x-axis at A and has a stationary point at B.

(a) State the coordinates of A. *(1 mark)*

(b) Find the coordinates of the stationary point, B, of the curve, giving your answer in an exact form. *(5 marks)*

(c) Find the exact value of the gradient of the normal to the curve at the point where $x = e^3$. *(3 marks)*

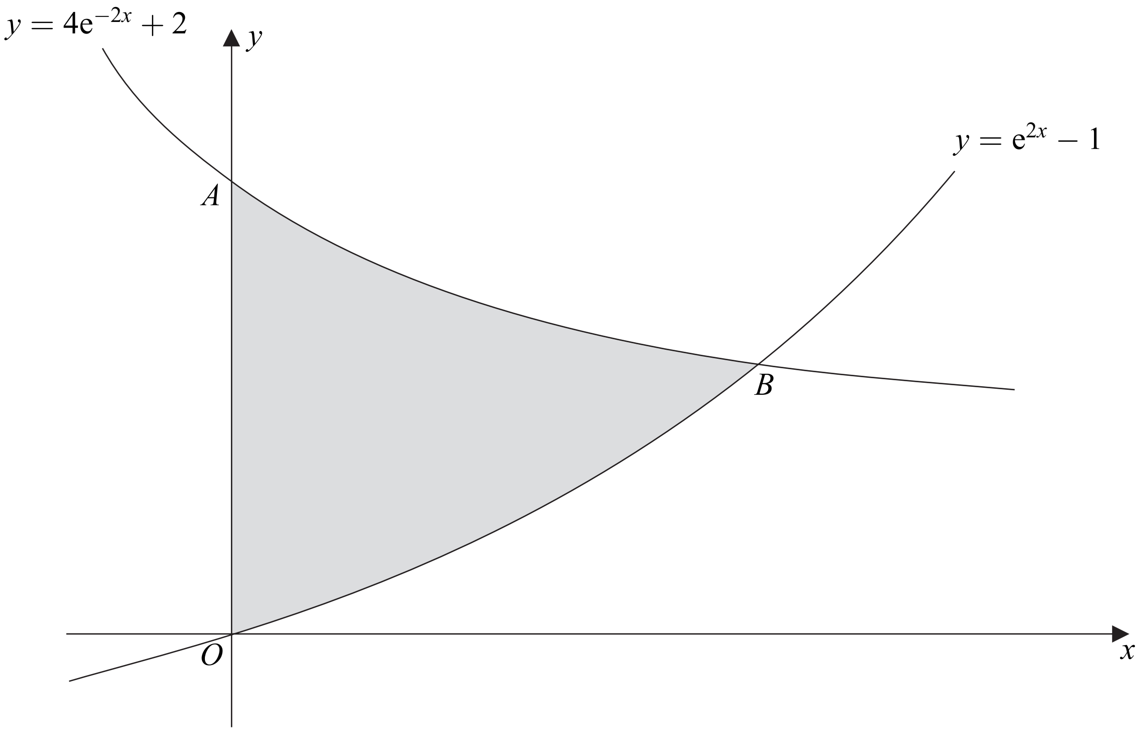
7 (a) Use integration by parts to find:

(i) $\int x \cos 4x \, dx$; *(4 marks)*

(ii) $\int x^2 \sin 4x \, dx$. *(4 marks)*

(b) The region bounded by the curve $y = 8x\sqrt{(\sin 4x)}$ and the lines $x = 0$ and $x = 0.2$ is rotated through 2π radians about the x-axis. Find the value of the volume of the solid generated, giving your answer to three significant figures. *(3 marks)*

8 The diagram shows the curves $y = e^{2x} - 1$ and $y = 4e^{-2x} + 2$.



The curve $y = 4e^{-2x} + 2$ crosses the y-axis at the point A and the curves intersect at the point B.

(a) Describe a sequence of two geometrical transformations that maps the graph of $y = e^{x}$ onto the graph of $y = e^{2x} - 1$. *(4 marks)*

(b) Write down the coordinates of the point A. *(1 mark)*

(c) (i) Show that the x-coordinate of the point B satisfies the equation

$$(e^{2x})^2 - 3e^{2x} - 4 = 0$$ *(2 marks)*

(ii) Hence find the exact value of the x-coordinate of the point B. *(3 marks)*

(d) Find the exact value of the area of the shaded region bounded by the curves $y = e^{2x} - 1$ and $y = 4e^{-2x} + 2$ and the y-axis. *(5 marks)*

END OF QUESTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MPC3

Unit Pure Core 3

Wednesday 19 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 (a) Find $\frac{dy}{dx}$ when $y = (x^3 - 1)^6$. *(2 marks)*

(b) A curve has equation $y = x \ln x$.

(i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the tangent to the curve $y = x \ln x$ at the point on the curve where $x = e$. *(3 marks)*

QUESTION PART REFERENCE

2 A curve is defined by the equation $y = (x^2 - 4)\ln(x + 2)$ for $x \geqslant 3$.

The curve intersects the line $y = 15$ at a single point, where $x = \alpha$.

(a) Show that α lies between 3.5 and 3.6. *(2 marks)*

(b) Show that the equation $(x^2 - 4)\ln(x + 2) = 15$ can be arranged into the form

$$x = \pm\sqrt{4 + \frac{15}{\ln(x + 2)}}$$ *(2 marks)*

(c) Use the iteration

$$x_{n+1} = \sqrt{4 + \frac{15}{\ln(x_n + 2)}}$$

with $x_1 = 3.5$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

3 (a) Given that $x = \tan(3y + 1)$:

(i) find $\dfrac{dx}{dy}$ in terms of y; *(2 marks)*

(ii) find the value of $\dfrac{dy}{dx}$ when $y = -\dfrac{1}{3}$. *(2 marks)*

(b) Sketch the graph of $y = \tan^{-1} x$. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

4 The functions f and g are defined with their respective domains by

$$f(x) = 3\cos\frac{1}{2}x, \qquad \text{for } 0 \leqslant x \leqslant 2\pi$$

$$g(x) = |x|, \qquad \text{for all real values of } x$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $f^{-1}(x) = 1$, giving your answer in an exact form. *(2 marks)*

(c) (i) Write down an expression for $gf(x)$. *(1 mark)*

(ii) Sketch the graph of $y = gf(x)$ for $0 \leqslant x \leqslant 2\pi$. *(3 marks)*

(d) Describe a sequence of two geometrical transformations that maps the graph of $y = \cos x$ onto the graph of $y = 3\cos\frac{1}{2}x$. *(3 marks)*

QUESTION PART REFERENCE

5 (a) Find $\int \frac{1}{3+2x}\,dx$. *(2 marks)*

(b) By using integration by parts, find $\int x \sin\frac{x}{2}\,dx$. *(4 marks)*

QUESTION PART REFERENCE

6 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^{0.4} \cos\sqrt{3x+1}\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) Use the substitution $u = 3x + 1$ to find the exact value of $\int_0^1 x\sqrt{3x+1}\,dx$. *(6 marks)*

QUESTION PART REFERENCE

7 (a) Solve the equation $\sec x = -5$, giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. *(3 marks)*

(b) Show that the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

can be written in the form

$$\sec^2 x = 25$$

(4 marks)

(c) Hence, or otherwise, solve the equation

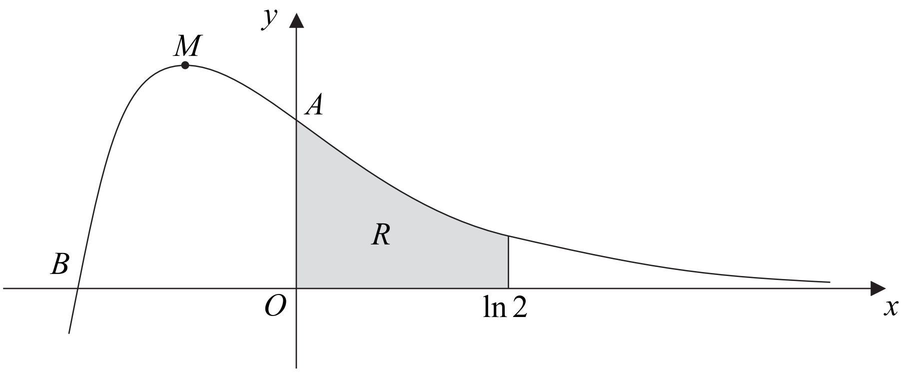
$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. *(3 marks)*

QUESTION PART REFERENCE

8 (a) Given that $e^{-2x} = 4$, find the exact value of x. *(2 marks)*

(b) The diagram shows the curve $y = 4e^{-2x} - e^{-4x}$.



The curve crosses the y-axis at the point A, the x-axis at the point B, and has a stationary point at M.

(i) State the y-coordinate of A. *(1 mark)*

(ii) Find the x-coordinate of B, giving your answer in an exact form. *(3 marks)*

(iii) Find the x-coordinate of the stationary point, M, giving your answer in an exact form. *(3 marks)*

(iv) The shaded region R is bounded by the curve $y = 4e^{-2x} - e^{-4x}$, the lines $x = 0$ and $x = \ln 2$ and the x-axis.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in the form $\frac{p}{q}\pi$, where p and q are integers. *(7 marks)*

QUESTION PART REFERENCE

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MPC3

Unit Pure Core 3

Monday 13 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

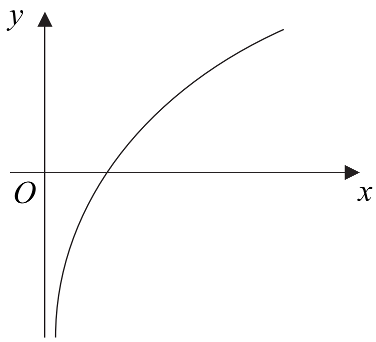
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The diagram shows the curve with equation $y = \ln(6x)$.



(a) State the x-coordinate of the point of intersection of the curve with the x-axis. *(1 mark)*

(b) Find $\dfrac{dy}{dx}$. *(2 marks)*

(c) Use Simpson's rule with 6 strips (7 ordinates) to find an estimate for $\int_1^7 \ln(6x)\,dx$, giving your answer to three significant figures. *(4 marks)*

---

2 (a) (i) Find $\dfrac{dy}{dx}$ when $y = xe^{2x}$. *(3 marks)*

(ii) Find an equation of the tangent to the curve $y = xe^{2x}$ at the point $(1, e^2)$. *(2 marks)*

(b) Given that $y = \dfrac{2\sin 3x}{1 + \cos 3x}$, use the quotient rule to show that

$$\frac{dy}{dx} = \frac{k}{1 + \cos 3x}$$

where k is an integer. *(4 marks)*

3 The curve $y = \cos^{-1}(2x - 1)$ intersects the curve $y = e^x$ at a single point where $x = \alpha$.

(a) Show that α lies between 0.4 and 0.5. *(2 marks)*

(b) Show that the equation $\cos^{-1}(2x - 1) = e^x$ can be written as $x = \frac{1}{2} + \frac{1}{2}\cos(e^x)$. *(1 mark)*

(c) Use the iteration $x_{n+1} = \frac{1}{2} + \frac{1}{2}\cos(e^{x_n})$ with $x_1 = 0.4$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

---

4 (a) (i) Solve the equation $\operatorname{cosec}\theta = -4$ for $0° < \theta < 360°$, giving your answers to the nearest 0.1°. *(2 marks)*

(ii) Solve the equation

$$2\cot^2(2x + 30°) = 2 - 7\operatorname{cosec}(2x + 30°)$$

for $0° < x < 180°$, giving your answers to the nearest 0.1°. *(6 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \operatorname{cosec} x$ onto the graph of $y = \operatorname{cosec}(2x + 30°)$. *(4 marks)*

---

5 The functions f and g are defined with their respective domains by

$$f(x) = x^2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x + 1} \qquad \text{for real values of } x, \quad x \neq -0.5$$

(a) Explain why f does not have an inverse. *(1 mark)*

(b) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

(c) State the range of g^{-1}. *(1 mark)*

(d) Solve the equation $fg(x) = g(x)$. *(3 marks)*

6 (a) Given that $3\ln x = 4$, find the exact value of x. *(1 mark)*

(b) By forming a quadratic equation in $\ln x$, solve $3\ln x + \dfrac{20}{\ln x} = 19$, giving your answers for x in an exact form. *(5 marks)*

---

7 (a) On separate diagrams:

(i) sketch the curve with equation $y = |3x + 3|$; *(2 marks)*

(ii) sketch the curve with equation $y = |x^2 - 1|$. *(3 marks)*

(b) (i) Solve the equation $|3x + 3| = |x^2 - 1|$. *(5 marks)*

(ii) Hence solve the inequality $|3x + 3| < |x^2 - 1|$. *(2 marks)*

---

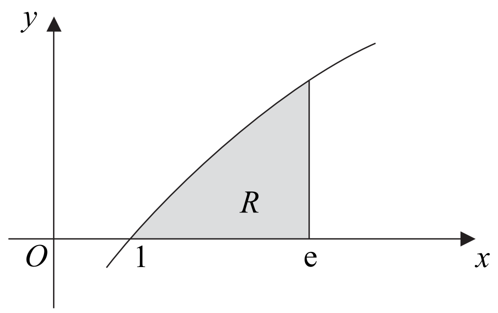
8 Use the substitution $u = 1 + 2\tan x$ to find

$$\int \frac{1}{(1 + 2\tan x)^2 \cos^2 x}\,\mathrm{d}x$$ *(5 marks)*

9 (a) Use integration by parts to find $\int x \ln x \, dx$. *(3 marks)*

(b) Given that $y = (\ln x)^2$, find $\frac{dy}{dx}$. *(2 marks)*

(c) The diagram shows part of the curve with equation $y = \sqrt{x} \ln x$.



The shaded region R is bounded by the curve $y = \sqrt{x} \ln x$, the line $x = e$ and the x-axis from $x = 1$ to $x = e$.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in an exact form. *(6 marks)*

END OF QUESTIONS

General Certificate of Education
Advanced Level Examination
January 2012

Mathematics MPC3

Unit Pure Core 3

Friday 20 January 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
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- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 (a) Use Simpson's rule with 7 ordinates (6 strips) to find an estimate for $\int_0^3 4^x \, dx$. *(4 marks)*

(b) A curve is defined by the equation $y = 4^x$. The curve intersects the line $y = 8 - 2x$ at a single point where $x = \alpha$.

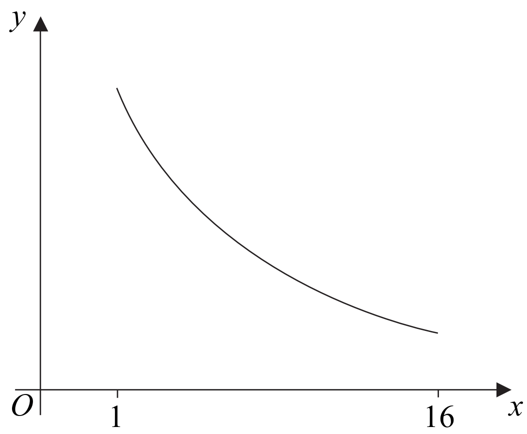
(i) Show that α lies between 1.2 and 1.3. *(2 marks)*

(ii) The equation $4^x = 8 - 2x$ can be rearranged into the form $x = \dfrac{\ln(8 - 2x)}{\ln 4}$.

Use the iterative formula $x_{n+1} = \dfrac{\ln(8 - 2x_n)}{\ln 4}$ with $x_1 = 1.2$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

---

2 The curve with equation $y = \dfrac{63}{4x - 1}$ is sketched below for $1 \leqslant x \leqslant 16$.



The function f is defined by $f(x) = \dfrac{63}{4x - 1}$ for $1 \leqslant x \leqslant 16$.

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $f^{-1}(x) = 1$. *(2 marks)*

(c) The function g is defined by $g(x) = x^2$ for $-4 \leqslant x \leqslant -1$.

(i) Write down an expression for $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 1$. *(3 marks)*

3 (a) Given that $y = 4x^3 - 6x + 1$, find $\dfrac{dy}{dx}$. *(1 mark)*

(b) Hence find $\displaystyle\int_2^3 \frac{2x^2 - 1}{4x^3 - 6x + 1}\,dx$, giving your answer in the form $p \ln q$, where p and q are rational numbers. *(5 marks)*

---

4 (a) By using a suitable trigonometrical identity, solve the equation

$$\tan^2 \theta = 3(3 - \sec \theta)$$

giving all solutions to the nearest 0.1° in the interval $0° < \theta < 360°$. *(6 marks)*

(b) Hence solve the equation

$$\tan^2(4x - 10°) = 3[3 - \sec(4x - 10°)]$$

giving all solutions to the nearest 0.1° in the interval $0° < x < 90°$. *(3 marks)*

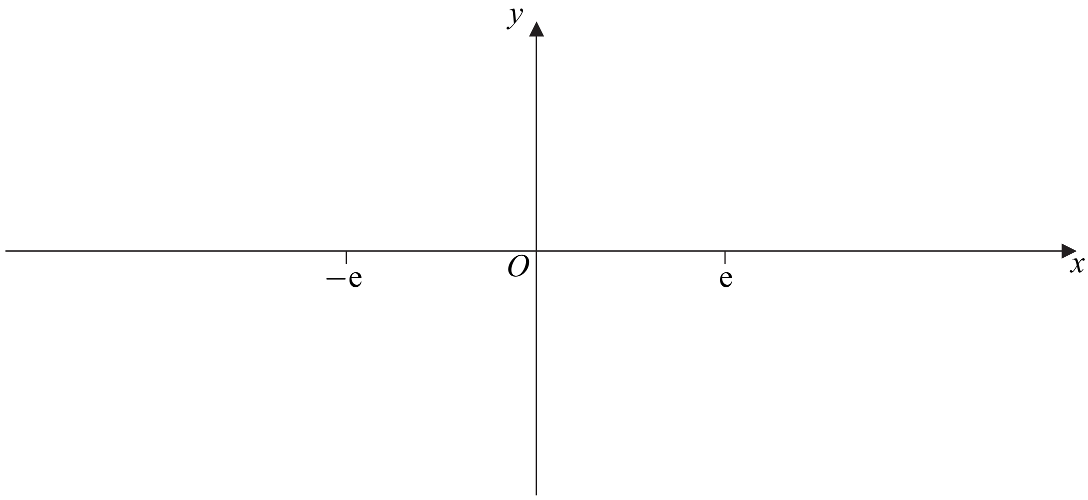
---

5 (a) Describe a sequence of two geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 4\ln(x - e)$. *(4 marks)*

(b) Sketch, on the axes given below, the graph of $y = |4\ln(x - e)|$, indicating the exact value of the x-coordinate where the curve meets the x-axis. *(3 marks)*

(c) (i) Solve the equation $|4\ln(x - e)| = 4$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|4\ln(x - e)| \geqslant 4$. *(3 marks)*



Turn over ▶

6 (a) Given that $x = \dfrac{1}{\sin\theta}$, use the quotient rule to show that $\dfrac{dx}{d\theta} = -\operatorname{cosec}\theta\cot\theta$. *(3 marks)*

(b) Use the substitution $x = \operatorname{cosec}\theta$ to find $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,dx$, giving your answer to three significant figures. *(9 marks)*

---

7 (a) A curve has equation $y = x^2 e^{-\frac{x}{4}}$.

Show that the curve has exactly two stationary points and find the exact values of their coordinates. *(7 marks)*

(b) (i) Use integration by parts twice to find the exact value of $\displaystyle\int_0^4 x^2 e^{-\frac{x}{4}}\,dx$. *(7 marks)*

(ii) The region bounded by the curve $y = 3xe^{-\frac{x}{8}}$, the x-axis from 0 to 4 and the line $x = 4$ is rotated through 360° about the x-axis to form a solid.

Use your answer to part **(b)(i)** to find the exact value of the volume of the solid generated. *(2 marks)*

General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MPC3

Unit Pure Core 3

Thursday 31 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Use the mid-ordinate rule with four strips to find an estimate for $\int_{0.4}^{1.2} \cot(x^2)\, dx$, giving your answer to three decimal places. *(4 marks)*

---

2 For $0 < x \leqslant 2$, the curves with equations $y = 4\ln x$ and $y = \sqrt{x}$ intersect at a single point where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.5. *(2 marks)*

(b) Show that the equation $4\ln x = \sqrt{x}$ can be rearranged into the form

$$x = e^{\left(\frac{\sqrt{x}}{4}\right)}$$

(1 mark)

(c) Use the iterative formula

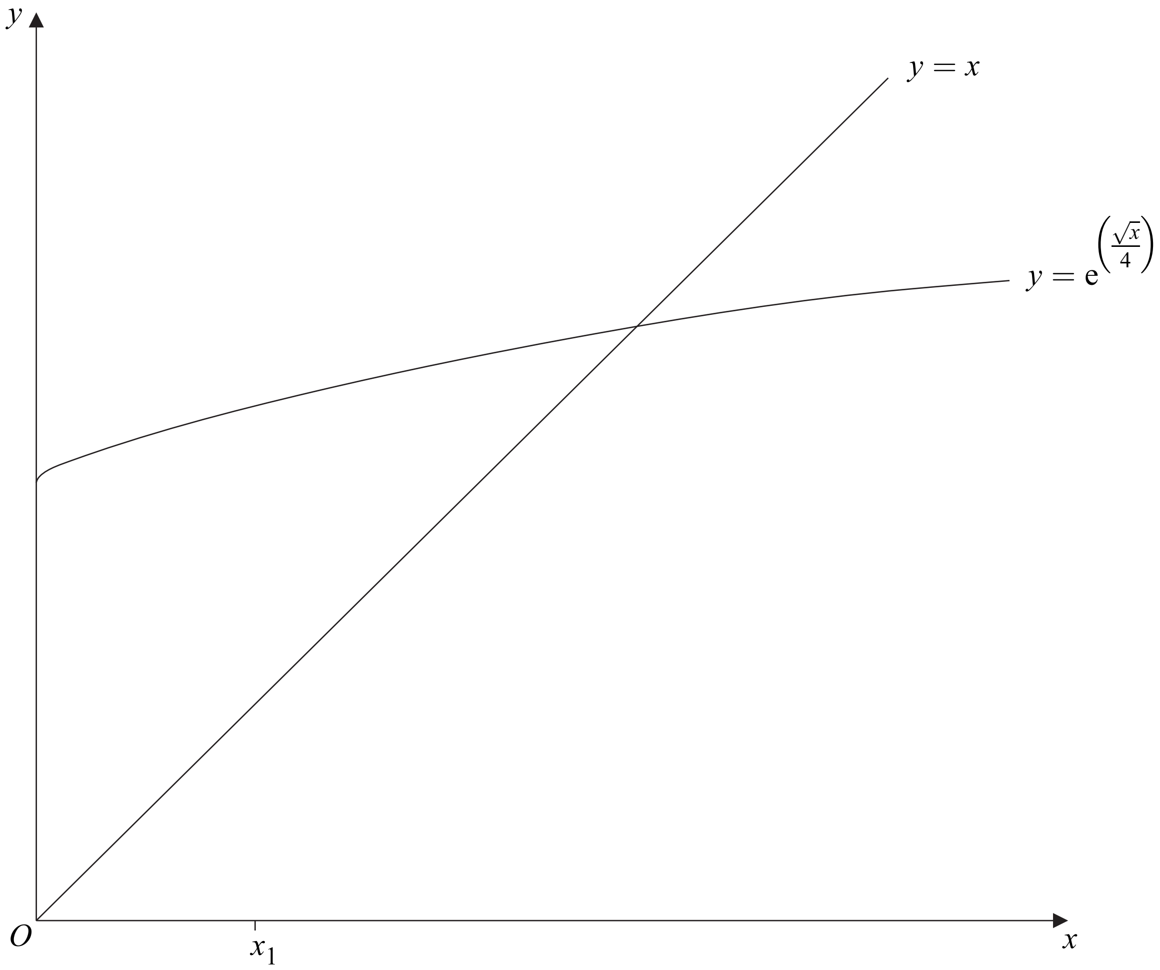
$$x_{n+1} = e^{\left(\frac{\sqrt{x_n}}{4}\right)}$$

with $x_1 = 0.5$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(d) **Figure 1**, on the page 3, shows a sketch of parts of the graphs of $y = e^{\left(\frac{\sqrt{x}}{4}\right)}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

Figure 1



---

3 A curve has equation $y = x^3 \ln x$.

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

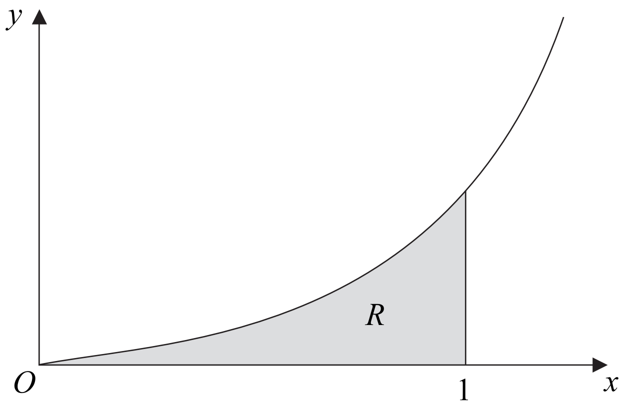
(b) (i) Find an equation of the tangent to the curve $y = x^3 \ln x$ at the point on the curve where $x = e$. *(3 marks)*

(ii) This tangent intersects the x-axis at the point A. Find the exact value of the x-coordinate of the point A. *(2 marks)*

Turn over ▶

4 (a) By using integration by parts, find $\int x\,\mathrm{e}^{6x}\,\mathrm{d}x$. *(4 marks)*

(b) The diagram shows part of the curve with equation $y = \sqrt{x}\,\mathrm{e}^{3x}$.



The shaded region R is bounded by the curve $y = \sqrt{x}\,\mathrm{e}^{3x}$, the line $x = 1$ and the x-axis from $x = 0$ to $x = 1$.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in the form $\pi(p\mathrm{e}^6 + q)$, where p and q are rational numbers. *(3 marks)*

---

5 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \sqrt{2x - 5}, \quad \text{for } x \geqslant 2.5$$

$$\mathrm{g}(x) = \frac{10}{x}, \quad \text{for real values of } x, \quad x \neq 0$$

(a) State the range of f. *(2 marks)*

(b) (i) Find $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 5$. *(2 marks)*

(c) The inverse of f is f^{-1}.

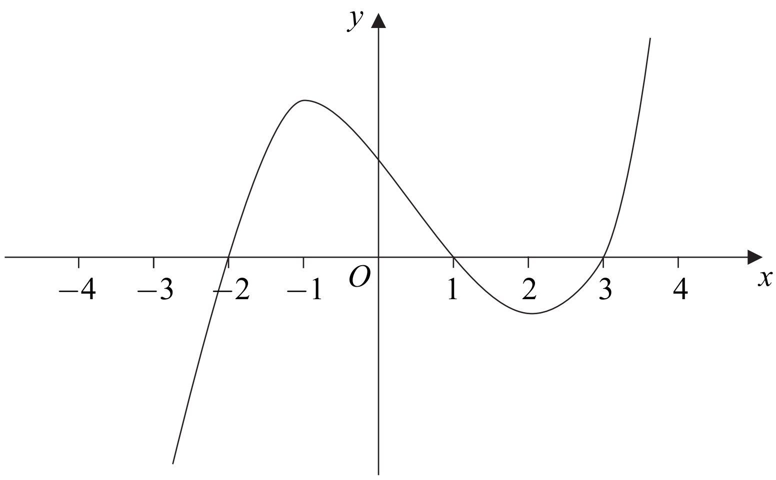
(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 7$. *(2 marks)*

6 Use the substitution $u = x^4 + 2$ to find the value of $\int_0^1 \frac{x^7}{(x^4+2)^2}\,dx$, giving your answer in the form $p \ln q + r$, where p, q and r are rational numbers. *(6 marks)*

---

7 The sketch shows part of the curve with equation $y = f(x)$.



(a) On **Figure 2** on page 6, sketch the curve with equation $y = |\,f(x)\,|$. *(3 marks)*

(b) On **Figure 3** on page 6, sketch the curve with equation $y = f(|\,x\,|)$. *(2 marks)*

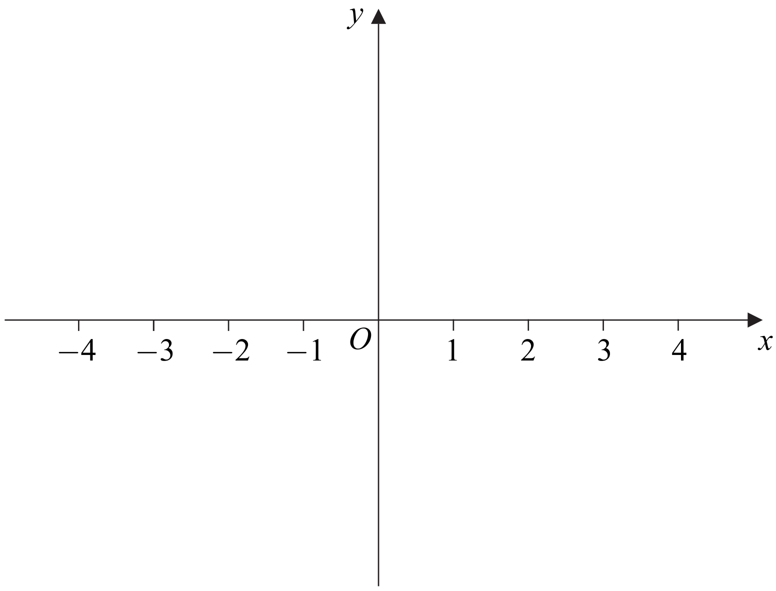
(c) Describe a sequence of two geometrical transformations that maps the graph of $y = f(x)$ onto the graph of $y = \frac{1}{2}f(x+1)$. *(4 marks)*

(d) The maximum point of the curve with equation $y = f(x)$ has coordinates $(-1, 10)$. Find the coordinates of the maximum point of the curve with equation $y = \frac{1}{2}f(x+1)$. *(2 marks)*

Turn over ▶

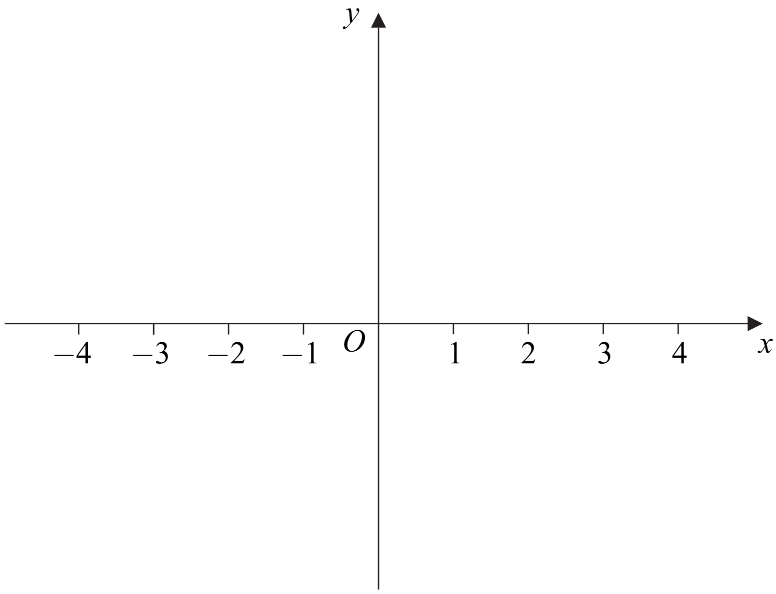
(a)

Figure 2



(b)

Figure 3



8 (a) Show that the equation

$$\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}=32$$

can be written in the form

$$\operatorname{cosec}^2\theta = 16$$ *(4 marks)*

(b) Hence, or otherwise, solve the equation

$$\frac{1}{1+\cos(2x-0.6)}+\frac{1}{1-\cos(2x-0.6)}=32$$

giving all values of x in radians to two decimal places in the interval $0 < x < \pi$.

(5 marks)

---

9 (a) Given that $x = \dfrac{\sin y}{\cos y}$, use the quotient rule to show that

$$\frac{dx}{dy} = \sec^2 y$$ *(3 marks)*

(b) Given that $\tan y = x - 1$, use a trigonometrical identity to show that

$$\sec^2 y = x^2 - 2x + 2$$ *(2 marks)*

(c) Show that, if $y = \tan^{-1}(x-1)$, then

$$\frac{dy}{dx} = \frac{1}{x^2 - 2x + 2}$$ *(1 mark)*

(d) A curve has equation $y = \tan^{-1}(x-1) - \ln x$.

(i) Find the value of the x-coordinate of each of the stationary points of the curve.

(4 marks)

(ii) Find $\dfrac{d^2y}{dx^2}$. *(2 marks)*

(iii) Hence show that the curve has a minimum point which lies on the x-axis. *(2 marks)*

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MPC3

Unit Pure Core 3

Wednesday 23 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Show that the equation $x^3 - 6x + 1 = 0$ has a root α, where $2 < \alpha < 3$. *(2 marks)*

(b) Show that the equation $x^3 - 6x + 1 = 0$ can be rearranged into the form

$$x^2 = 6 - \frac{1}{x}$$ *(1 mark)*

(c) Use the recurrence relation $x_{n+1} = \sqrt{6 - \frac{1}{x_n}}$, with $x_1 = 2.5$, to find the value of x_3, giving your answer to four significant figures. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

2 (a) Use Simpson's rule, with five ordinates (four strips), to calculate an estimate for

$$\int_0^4 \frac{x}{x^2+2}\,\mathrm{d}x$$

Give your answer to four significant figures. *(4 marks)*

(b) Show that the exact value of $\int_0^4 \frac{x}{x^2+2}\,\mathrm{d}x$ is $\ln k$, where k is an integer. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 2

3 (a) Find $\dfrac{dy}{dx}$ when

$$y = e^{3x} + \ln x$$ *(2 marks)*

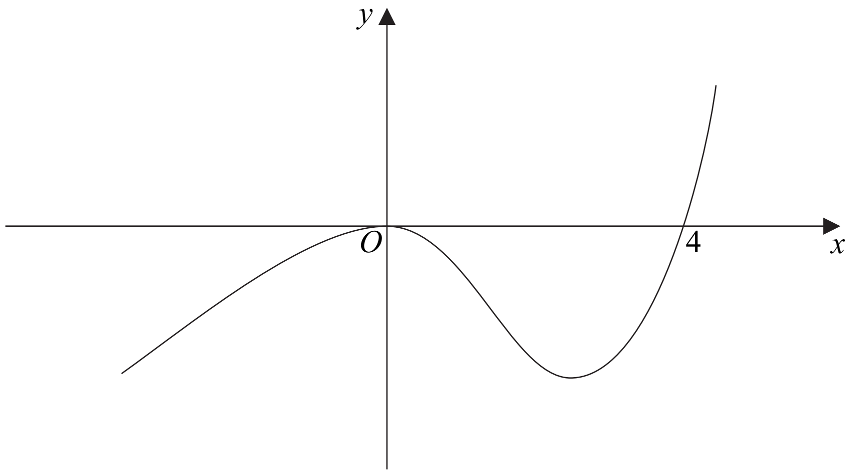
(b) (i) Given that $u = \dfrac{\sin x}{1 + \cos x}$, show that $\dfrac{du}{dx} = \dfrac{1}{1 + \cos x}$. *(3 marks)*

(ii) Hence show that if $y = \ln\left(\dfrac{\sin x}{1 + \cos x}\right)$, then $\dfrac{dy}{dx} = \operatorname{cosec} x$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 3

4 The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$.



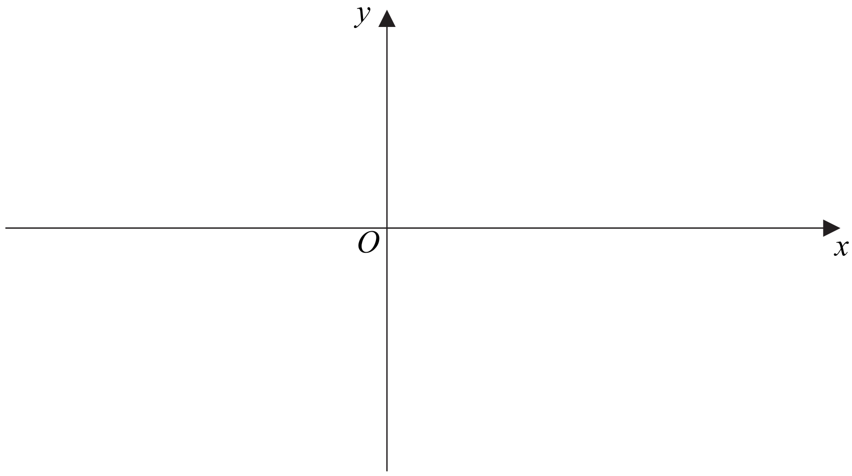
(a) On the axes below, sketch the curve with equation $y = |\mathrm{f}(x)|$. *(2 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}(2x - 1)$. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 4

(a)



5 The function f is defined by

$$f(x) = \frac{x^2 - 4}{3}, \quad \text{for real values of } x, \textbf{ where } \boldsymbol{x \leqslant 0}$$

(a) State the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

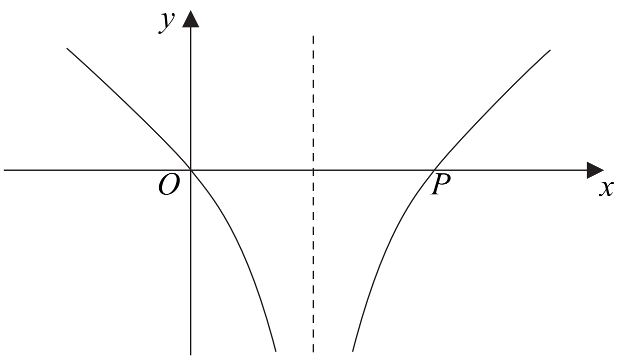
(i) Write down the domain of f^{-1}. *(1 mark)*

(ii) Find an expression for $f^{-1}(x)$. *(3 marks)*

(c) The function g is defined by

$$g(x) = \ln|3x - 1|, \quad \text{for real values of } x, \text{ where } x \neq \frac{1}{3}$$

The curve with equation $y = g(x)$ is sketched below.



(i) The curve $y = g(x)$ intersects the x-axis at the origin and at the point P.

Find the x-coordinate of P. *(2 marks)*

(ii) State whether the function g has an inverse. Give a reason for your answer. *(1 mark)*

(iii) Show that $gf(x) = \ln|x^2 - k|$, stating the value of the constant k. *(2 marks)*

(iv) Solve the equation $gf(x) = 0$. *(4 marks)*

6 (a) Show that

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)}$$

can be written as $\operatorname{cosec}^2 x$. *(3 marks)*

(b) Hence solve the equation

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)} = \operatorname{cosec} x + 3$$

giving the values of x to the nearest degree in the interval $-180° < x < 180°$. *(6 marks)*

(c) Hence solve the equation

$$\frac{\sec^2(2\theta - 60°)}{(\sec(2\theta - 60°) + 1)(\sec(2\theta - 60°) - 1)} = \operatorname{cosec}(2\theta - 60°) + 3$$

giving the values of θ to the nearest degree in the interval $0° < \theta < 90°$. *(2 marks)*

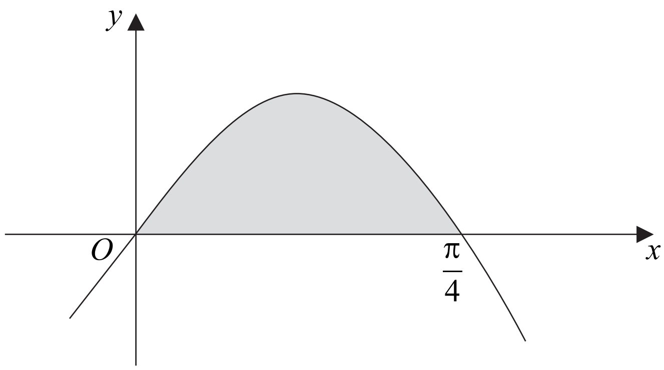
QUESTION PART REFERENCE

Answer space for question 6

7 A curve has equation $y = 4x \cos 2x$.

(a) Find an exact equation of the tangent to the curve at the point on the curve where $x = \frac{\pi}{4}$. *(5 marks)*

(b) The region shaded on the diagram below is bounded by the curve $y = 4x \cos 2x$ and the x-axis from $x = 0$ to $x = \frac{\pi}{4}$.



By using integration by parts, find the exact value of the area of the shaded region. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

8 (a) Show that

$$\int_0^{\ln 2} e^{1-2x}\, dx = \frac{3}{8}e$$

(4 marks)

(b) Use the substitution $u = \tan x$ to find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^4 x \sqrt{\tan x}\, dx$$

(8 marks)

QUESTION PART REFERENCE

Answer space for question 8

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MPC3

Unit Pure Core 3

Thursday 6 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
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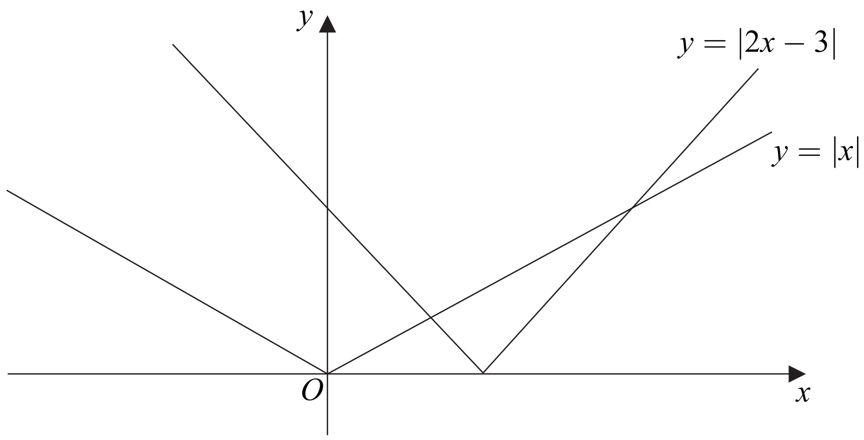
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The diagram below shows the graphs of $y = |2x - 3|$ and $y = |x|$.



(a) Find the x-coordinates of the points of intersection of the graphs of $y = |2x - 3|$ and $y = |x|$. *(3 marks)*

(b) Hence, or otherwise, solve the inequality

$$|2x - 3| \geqslant |x|$$ *(2 marks)*

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2 (a) Given that $y = x^4 \tan 2x$, find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Find the gradient of the curve with equation $y = \dfrac{x^2}{x - 1}$ at the point where $x = 3$. *(3 marks)*

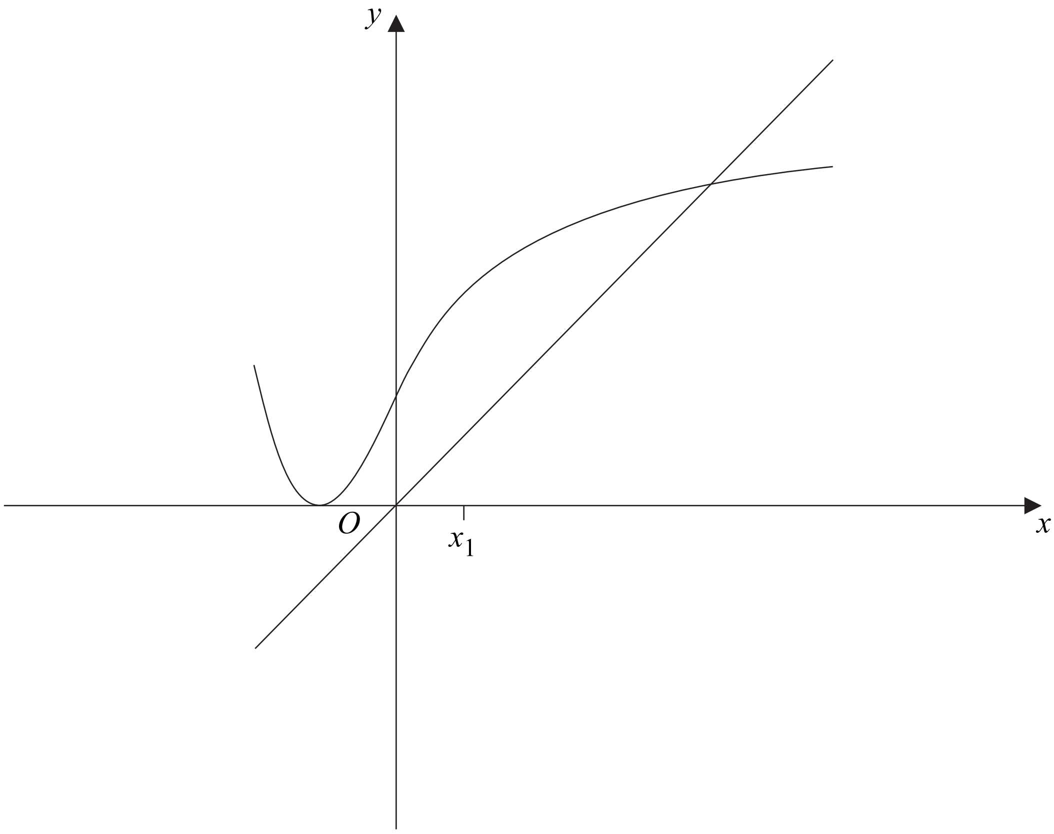
3 (a) The equation $e^{-x} - 2 + \sqrt{x} = 0$ has a single root, α.

Show that α lies between 3 and 4. *(2 marks)*

(b) Use the recurrence relation $x_{n+1} = (2 - e^{-x_n})^2$, with $x_1 = 3.5$, to find x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(c) The diagram below shows parts of the graphs of $y = (2 - e^{-x})^2$ and $y = x$, and a position of x_1.

On the diagram, draw a staircase or cobweb diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis.



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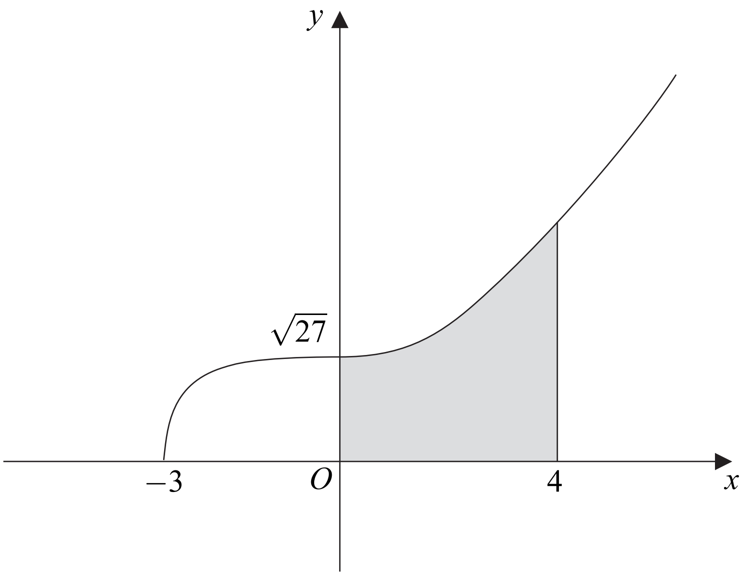
4 By forming and solving a quadratic equation, solve the equation

$$8 \sec x - 2 \sec^2 x = \tan^2 x - 2$$

in the interval $0 < x < 2\pi$, giving the values of x in radians to three significant figures. *(7 marks)*

Turn over ▶

5 The diagram shows a sketch of the graph of $y = \sqrt{27 + x^3}$.



(a) The area of the shaded region, bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$, is given by $\int_0^4 \sqrt{27 + x^3}\, dx$.

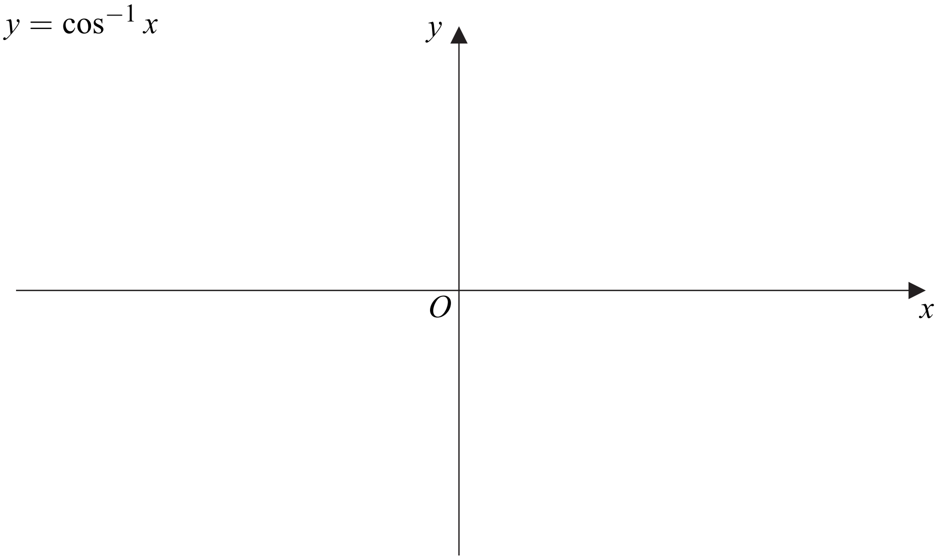
Use the mid-ordinate rule with **five** strips to find an estimate for this area. Give your answer to three significant figures. *(4 marks)*

(b) With the aid of a diagram, explain whether the mid-ordinate rule applied in part **(a)** gives an estimate which is smaller than or greater than the area of the shaded region. *(2 marks)*

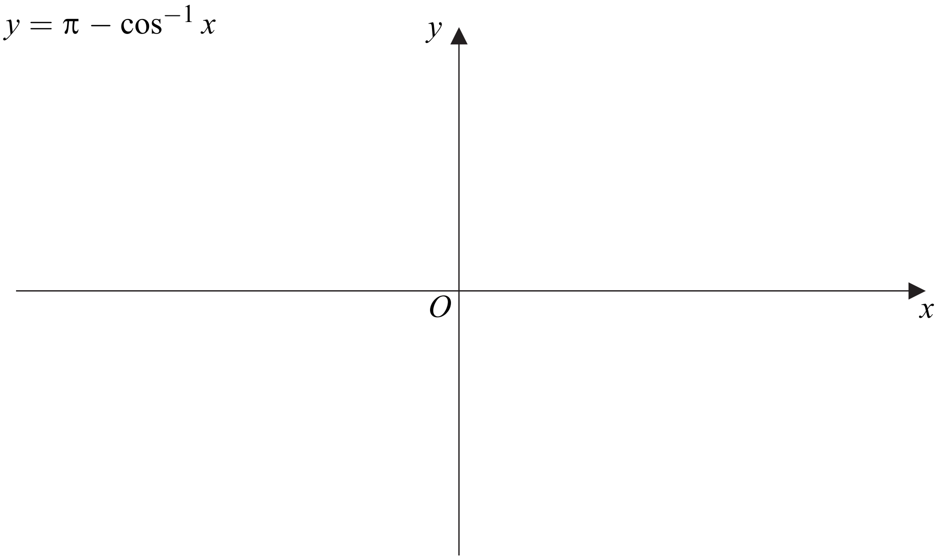
6 (a) Sketch the graph of $y = \cos^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. *(2 marks)*

(b) Sketch the graph of $y = \pi - \cos^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. *(2 marks)*

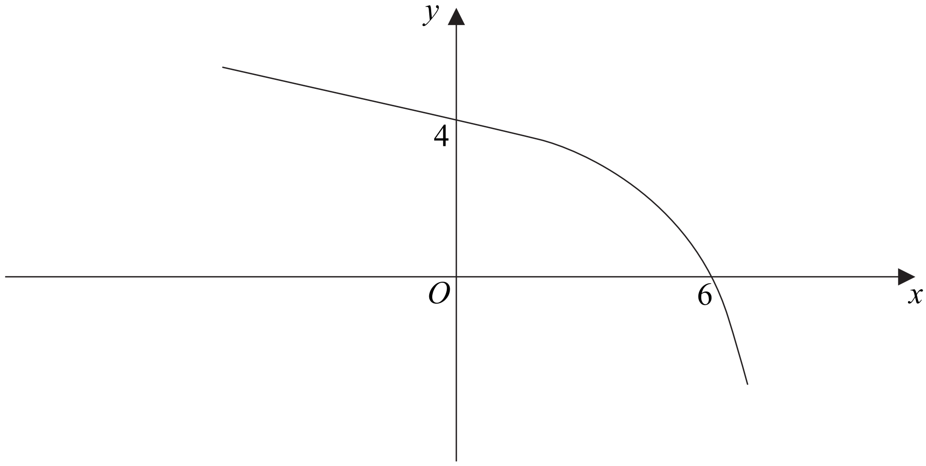
$y = \cos^{-1} x$



$y = \pi - \cos^{-1} x$



7 The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$.



(a) On **Figure 1**, below, sketch the curve with equation $y = -\mathrm{f}(3x)$, indicating the values where the curve cuts the coordinate axes. *(2 marks)*

(b) On **Figure 2**, on page 7, sketch the curve with equation $y = \mathrm{f}(|x|)$, indicating the values where the curve cuts the coordinate axes. *(3 marks)*

(c) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}\left(-\frac{1}{2}x\right)$. *(4 marks)*

Figure 1

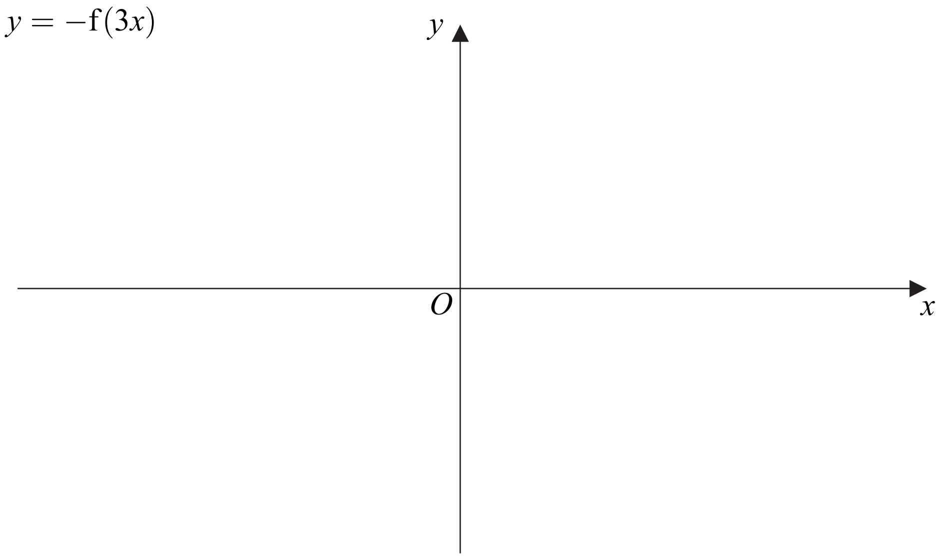
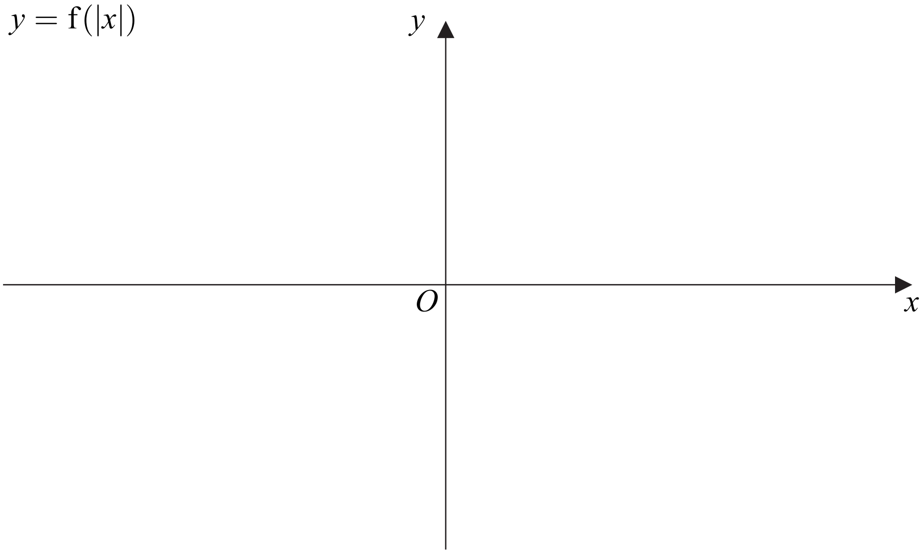
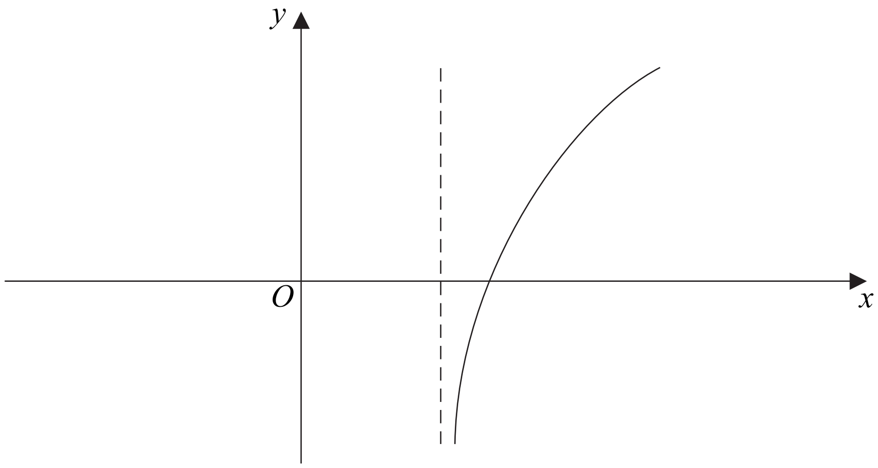


Figure 2



8 The curve with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \ln(2x - 3)$, $x > \frac{3}{2}$, is sketched below.



(a) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) State the range of f^{-1}. *(1 mark)*

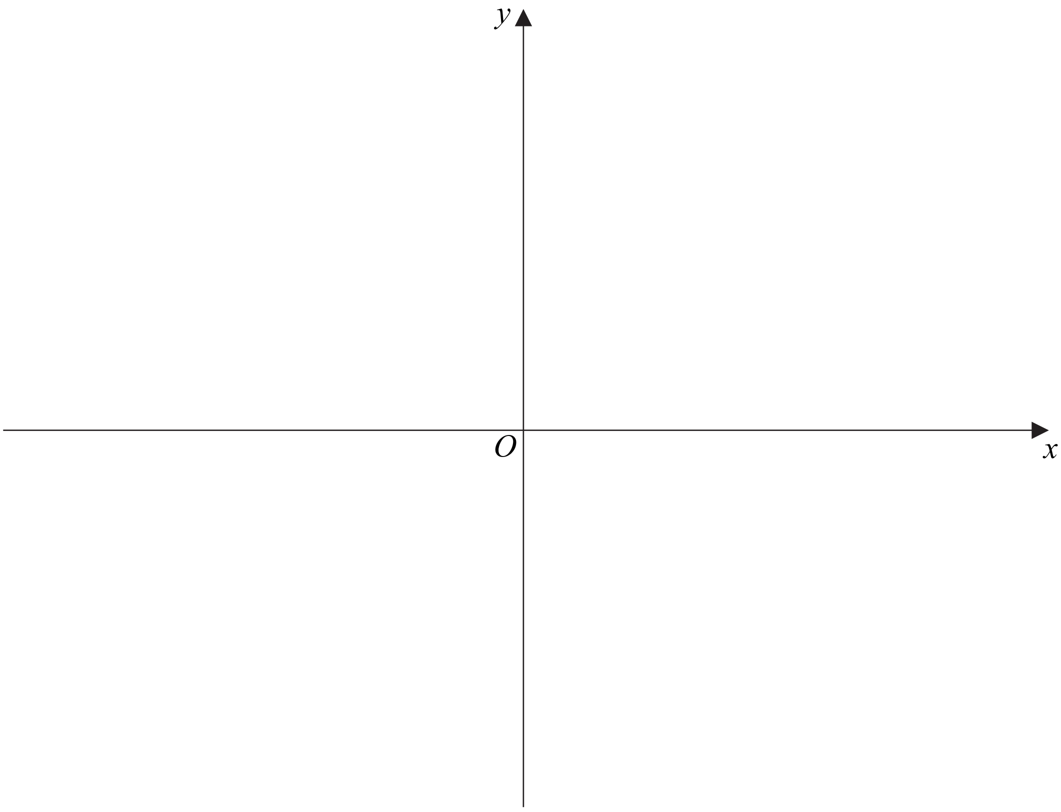
(iii) Sketch, on the axes given on page 9, the curve with equation $y = \mathrm{f}^{-1}(x)$, indicating the value of the y-coordinate of the point where the curve intersects the y-axis. *(2 marks)*

(b) The function g is defined by

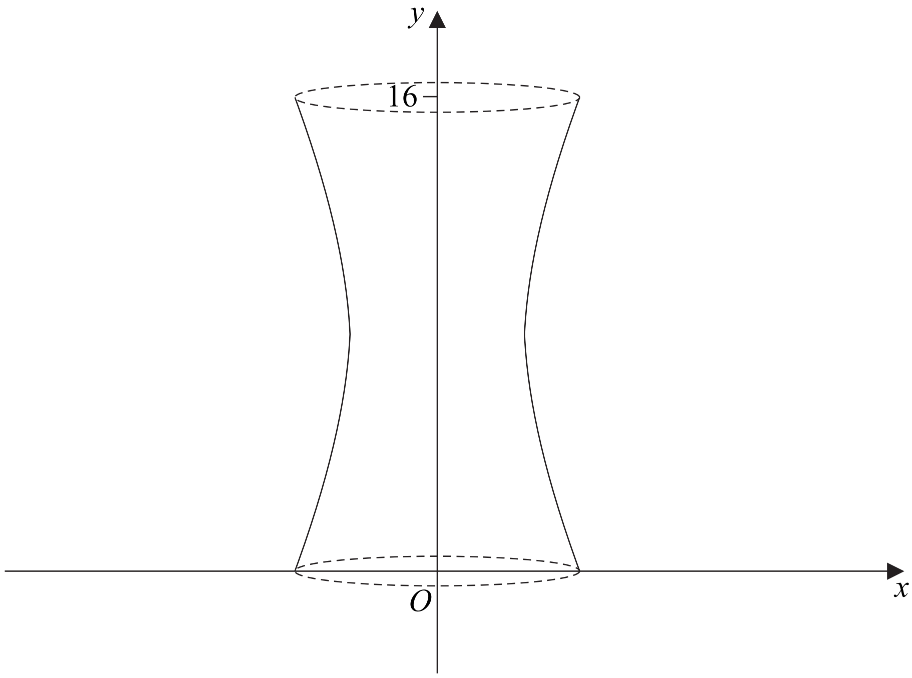
$$\mathrm{g}(x) = \mathrm{e}^{2x} - 4, \quad \text{for all real values of } x$$

(i) Find $\mathrm{gf}(x)$, giving your answer in the form $(ax - b)^2 - c$, where a, b and c are integers. *(3 marks)*

(ii) Write down an expression for $\mathrm{fg}(x)$, and hence find the exact solution of the equation $\mathrm{fg}(x) = \ln 5$. *(3 marks)*



9 The shape of a vase can be modelled by rotating the curve with equation $16x^2 - (y-8)^2 = 32$ between $y = 0$ and $y = 16$ completely **about the *y*-axis**.



The vase has a base.

Find the volume of water needed to fill the vase, giving your answer as an exact value. *(5 marks)*

10 (a) (i) By writing $\ln x$ as $(\ln x) \times 1$, use integration by parts to find $\int \ln x \, dx$. *(4 marks)*

(ii) Find $\int (\ln x)^2 \, dx$. *(4 marks)*

(b) Use the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{1}{x + \sqrt{x}} \, dx$$ *(7 marks)*